Investigation of Deterministic Dynamics in Hydrologic variables: A chaotic Perspective

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CERTIFICATE

It is certified that the work contained in the thesis entitled " **Investigation of Deterministic Dynamics in Hydrologic variables: A Chaotic Perspective**", by **R.SATISH KUMAR** (Roll No. Y010333) has been carried out under my supervision and this work has not been submitted elsewhere for any degree.

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ABSTRACT

The Present study carried out an extensive investigation of deterministic dynamics in hydrologic variables from a chaotic perspective. The hydrologic variables investigated include total daily rainfall and average daily flow. The data derived from Kentucky River Basin (KRB) and North Fork Kentucky (NFKY) River Basin, a part of the KRB, were employed in this study. Correlation Integral method and Lyapunov Exponent method were employed in this study for investigation of chaos. The Lyapunov Exponent was calculated over a consistency range, probably for the first time, as suggested by Rodriguez-Iturbe *et al.*, (1989).

The results obtained in the current study in terms of the Lyapunov Exponent strongly support the existence of chaos in both rainfall and flow data at all the locations investigated in the present study. Correlation Exponent results strongly supported the existence of chaos in the average daily flow, and the existence of low dimensional deterministic dynamics can not be ruled out in the daily rainfall data at all the locations considered in the present study, according to Porporato and Ridolfi (1996). The Present study also made an attempt to investigate for the temporal and spatial effects in both rainfall and flow data from a chaotic perspective. The temporal scaling effects in rainfall data were investigated for one, two, five, and seven days; whereas, the scaling effects in inflow data were investigated for one, five, and seven-day resolutions. The scaling effects for the river flow process have been found to be chaotic; and the existence of low dimensional chaotic dynamics in the rainfall process can not be ruled out, based on the results obtained for temporal scaling effects in this study. The Lyapunov Exponent

behaves in a manner similar to the coefficient of variation, but Correlation Exponent behaves in a contrary to the coefficient of variation, with respective to temporal scaling effects. Further, it has also been found that the nature of deterministic dynamics is space – independent for both rainfall and flow data.

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Chapter 1

INTRODUCTION

1.1 General

Hydrology is a science, which deals with modeling of various components of the hydrologic cycle, a key component of which is the rainfall-runoff process. During the past few decades, many models have been developed, and statements such as Linear v/s Non-linear, Deterministic v/s Stochastic, and Conceptual v/s Black box models became common vocabulary to the hydrologists. In addition to the above mentioned models in the last few years, Neural Networks have also been extensively employed for modeling various hydrologic processes. Recent studies on hydrologic modeling have concentrated in a different way *i.e.* finding the features of the dynamic nature of the underlying physical process. However, still there is no unified approach to modeling the hydrologic processes as these were influenced by many factors.

The hydrologic process is a dynamic, non-linear, and extremely complex physical process, which is affected by many inter-connected physical variables, making its modeling an extremely difficult task. The high temporal and spatial variations of the variables are also important in increasing the complexity of modeling. Noise and finite amount of the real data are limiting factors of the modeling techniques. Another part of the difficulty of hydrologic modeling is the unavailability of appropriate mathematical model to exploit the structure of the process. All these factors have made modeling of a hydrologic process more complex.

Modeling is a process in which we develop a relationship between output and the explanatory variables of a physical process. We can build the relation on the assumption of either a deterministic relation or a stochastic relation among the variables. A system is said to be deterministic if knowledge of the time evolutions, the parameters that describe the system, and the initial conditions in principle completely determine the subsequent behavior of the system. Hence, for deterministic systems, long-term prediction is feasible. In stochastic system, the input parameters are, in general, unknown or only statistical measures of the parameters are known. For such systems, even short-term prediction is not guaranteed.

The hydrologic processes, such as rainfall or stream flow, show large deviations from it's mean similar to those exhibited by stochastic processes. This type of behavior may result either from a random probabilistic structure or from a non-linear deterministic system highly sensitive to the initial conditions. Stochastic models are developed assuming that the random probabilistic nature prevails in the data. Though the stochastic technique is widely used in the modeling of rainfall and runoff, the assumptions made in it are contrary to the properties of the real system. The results of stochastic models are valid in the statistical sense only even though prediction in large time scale is possible. In the second case, the underlying dimension is perfectly deterministic and non linear, although appearance is similar to that of a stochastic process. The recent interest in non-linear dynamics and a rapidly growing set of tools for non-linear time series analysis has provided new ideas into the working of many such complex processes. This theory has gained considerable importance and has evolved into a phenomenon called "Chaos".

The term 'chaos' is used to refer to the irregular behavior of a dynamic system arising from a strictly deterministic time evolution without any source of noise or external stochasticity but with sensitivity to the initial conditions. In other words, small perturbations in the initial conditions have large effects in the future. Unlike stochastic systems, chaotic systems will have no random or unpredictable input or parameters. Even though the system looks in disorder inherently, it will have an order due to determinism. This interesting mixture of irregularity and order requires a different approach in studying complex processes like rainfall and runoff that are thought to be unpredictable. For deterministically chaotic systems, prediction in the short time scale is possible.

In many applications, non-linear modeling tools have provided better results when used in hydrologic time series analysis. Few example, among others, the superiority of artificial neural networks (ANN) over non linear regression in predicting river flows has been attributed to the possible existence of non linear dynamics, which was not captured by regression techniques (Elshorbagy *et al.*, 2000). Chaotic systems cannot be distinguished from stochastic processes using conventional statistical tools. Chaotic nature is investigated through the use of several techniques such as Correlation integral, Lyapunov Exponent, Fractal dimensions, and Kolomogorov entropy methods. Through these methods, one can find dynamical and geometrical aspects of the trajectories, which will quantify the chaos.

Further, the availability of high-resolution precise data is limited in the real world due to various reasons, cost being the major one, which are normally important inputs to various water- resources management applications. However, low-resolution

data can be easily made available in most cases. There fore, researchers have focussed their attention on data transformation not only from one temporal scale to the other but also over space. Investigation of temporal and spatial scaling effects from a chaotic perspective is in its infancy and extensive research efforts are needed in this area to understand these scaling effects. Such research efforts can reveal the dynamic nature of the transformation of data from one scale to other, and hence the suitable model for transformation of data.

1.2 Objectives of the Thesis

The objectives of the present work are manifold. Some of the objectives are listed below.

1. The main objective of the present work is to investigate for the existence of chaos in various hydrologic variables. Correlation Integral method will be used to analyze geometric aspects and Lyapunov Exponent method will be employed for dynamic aspects of a trajectory or a physical process.
2. The secondary objective of the present work is to investigate the scaling effects in the hydrologic variables at different time resolutions from a chaotic perspective.
3. The third objective of the present work is to investigate for spatial variation in hydrologic variables from a chaotic perspective.
4. The fourth objective of the present work is to investigate for any inter-relationships or trends among various standard statistical parameters (e.g. coefficient of variation) and several special measures of chaos (e.g. Correlation Exponent and Lyapunov Exponent) in a physical process.

The rainfall and runoff data from Kentucky River Basin (KRB) for a period of 30 years, and rainfall data from North Fork Kentucky (NFKY) River Basin (a part of the KRB) for 10 years were employed to carry out all investigations in this study.

1.3 Scope of Organization

The structure of the thesis is as follows: Chapter 1 provides general introduction to the topic, objectives of the thesis and organization of the work. Chapter 2 reviews the literature available in the area of chaos with special reference to hydrology. The basic concepts of chaos and its quantifying techniques are presented in the chapter 3. Chapter 4 describes the methodologies to quantify the chaos. In chapter 5, results and discussions are analyzed. In chapter 6, concluding remarks have been made. References and Appendices have been provided at the end.

Chapter 2

LITERATURE REVIEW

This chapter provides the information of the research that has been reported in literature in the area of the investigation of chaos and it's quantifying techniques. This chapter has been sectioned into two parts. First part describes the literature on the methods of the chaos quantifiers and the subsequent part describes the previously done work with the application of chaos in the field of hydrology.

2.1 Literature on Quantifying Chaos

Grassberger and Procaccia (1983) proposed a measure to characterize chaos and to distinguish a noisy random behavior from strictly deterministic chaos. They proposed an algorithm for the computation of correlation exponent of a uni-variate time series. The proposed algorithm has been used extensively for the purpose of investigation of chaos in a time series due to its simplicity. Correlation exponent can be taken as the most useful measure of the local structure of the strange attractor that demonstrates whether the system is chaotic or stochastic.

Grassberger and Procaccia (1983c) proposed a new method which can quantify the chaos in a given time series. They defined a new quantity K_2, which estimates the Kolmogorov Entropy (K) for a given time series. Till the introduction of the K_2, the calculation of the Kolmogorov Entropy (K) was tedious. The validity of Kolmogorov Entropy (K_2) has been tested on the known chaotic systems like Henon map and Mackey-

Glass differential equations. The results supported the existence of the chaos with positive K_2 value.

Wolf *et al.*, (1985) proposed two algorithms to calculate the Lyapunov Exponents for time series. First method is for defined modeled systems and the second method is for any observed time series. The first proposed algorithm was tested on model systems such as Lorenz attractor and Henon attractor with known lyapunov spectra. The second algorithm was applied to the experimental time series Belousov-Zhabotinskii reaction and Couette-Taylor flow. The application of the second algorithm to these two time series also showed positive Lyapunov Exponent reporting chaotic dynamics. Till these new developed algorithms were proposed by Wolf *et al.*, (1985), the calculation of Lyapunov Exponent using original technique was very tedious. Now, the methods proposed by wolf *et al.*, (1985), are widely used due to their simplicity in the computation of Lyapunov Exponent.

Unlike the above studies, Casdagli (1989) attempted to construct a predictive model directly from the time series. This method is known as nonlinear prediction method. He treated prediction as "inverse problem" in dynamical system. The results obtained by this method on the Lorenz equations and the Mackey-Glass delay differential equations showed encouraging results. Identification of the chaos can also be done by this method.

Kennel *et al.*, (1992) method proposed a new method called the method false neighbors to calculate the minimum embedding dimension for the phase space.

Minimum embedding dimension values obtained by this method on the Lorenz systems, Henon attractor, and Rossler attractor agree with the original values of the respective systems. It can be applied on the experimental data also.

2.2 Literature on Application of Theory of Chaos in Hydrology

The discovery of chaotic behavior in complex and non linear hydrologic processes stimulated researchers in the last few years, to investigate the existence of chaotic behavior in hydrology. The above-mentioned methods, namely, Correlation Dimension and Lyapunov Exponent are widely used in the detection of the chaos.

Rodriguez-Iturbe *et al.*, (1989) were probably the first ones to investigate the existence of chaos in hydrologic variables by employing both the Correlation dimension method and the Lyapunov exponent method. They analyzed the two rainfall events: 1) A record of 1990 rainfall observations of a storm measured at 15-s interval in Boston, and 2) Weekly rainfall data observed over a period of 148 years in Genoa, Italy. Observations of a finite low correlation dimension of about 3.78 and positive Lyapunov Exponent (0.0002 bits/sec) provided primary evidence on the existence of chaos in the storm-rainfall data. However weekly rainfall data did not show any indications supporting the presence of chaos.

Sharif *et al.*, (1990) analyzed three different rainfall records of storms with 4,000, 3,991 and 3,361 data points, respectively using Correlation Dimension method. Saturated lower values of the correlation exponent between 3 and 4 showed the existence of chaos in the storm rainfall observations with a low dimensional strange attractor.

Jaiwardena and Lai (1994) investigated the daily rainfall and stream flow observations from three stations and two stations, respectively, in Hong Kong for the purpose of identifying the existence of chaos. Correlation dimension, Lyapunov Exponent, Kolomogrov entropy, and non-linear prediction methods were employed to diagnose the chaos in the daily rainfall and stream flow observations of varying record lengths. The reliability of these techniques that characterize chaotic time series were verified using the artificially generated data from random, auto regressive moving average (ARMA) and chaotic series with additive noise. Results provided convincing evidence for the presence of chaos in the daily rainfall and stream flow data in Hong Kong. Their study demonstrated that rainfall and stream flow data series could be better modeled by time embedding method rather than the traditional linear ARMA approach.

Porporato and Ridolfi (1996) provided the clues to the existence of deterministic chaos in the river flow. The time series employed in this study had 14,246 observations of the daily river flow measured at Dora Balton, a tributary of the river Po, in Italy. They employed Correlation Integral and Non-linear prediction methods for the investigation of chaos. Then from the Correlation Integral results, Correlation Exponent was calculated for different embedding dimensions. Generally, Saturation of the Correlation Exponent after a particular embedding dimension will be considered as a supporting factor for the existence of chaos. However according to Porporato and Rudolfi (1996) "Even though a real plateau in the correlation exponent plateau does not exist and there is no complete saturation, such a behavior of the correlation integral allows the possibility that a low dimensional dynamics might be present in the phenomenon." Based

upon the above assumption they analyzed the Correlation Integration results. Results of the Correlation Integral and non-linear prediction method presented the evidence against the existence of deterministic component. This work paved the way to carry out further research in the field of noise reduction, interpolation and non-linear prediction method (Porporato and Ridolfi, 1997). They forecasted the stream flow values using non-linear prediction method, which produced remarkable predictive results. Wan and Gan (1998) also showed the existence of chaos in the unregulated stream flows. They employed the correlation integral method on the six rivers in the Canadian prairies and results presented low dimensional attractor of ~3.0.

Sivakumar *et al.*, (1998) investigated the existence of chaos in daily rainfall data from six rainfall stations in Singapore. Correlation integral method reported the presence of low dimensional attractor giving evidence for the existence of chaos. Sivakumar *et al.*, (1999) studied the daily rainfall data of different record lengths by varying the delay time for the purpose of identification of chaos in rainfall data. Correlation integral method and non-linear prediction method showed the existence of chaotic dynamics in the rainfall. Subsequent studies of Sivakumar *et al.*, (1999b and 1999c) concentrated on the influence of noise in the estimation of correlation integral and prediction estimates by a systematic approach for the noise reduction. The outcomes presented support the existence of deterministic component in the rainfall phenomenon and possible reasons for the low prediction accuracy estimates achieved in the earlier study (i.e. Sivakumar *et al.*, 1999a).

Stehlik (1999) analyzed the two runoff series from the experimental basin Jizerka in the Jizerka Mountains, Czech Republic. He used the Correlation Integral method to detect chaos in the daily runoff series and the runoff series with the 30-minute time interval. Results reported the low dimensional attractor with a fractal dimension of 2.94 in the 30-minute time interval runoff time series supporting chaotic dynamics. Contrary to this, the daily runoff time series was indistinguishable from a random process and therefore the deterministic dynamics could not be supported by the results for daily runoff series.

Stehlik (2000) investigated the precipitation and temperature time series with sampling intervals of 30 minutes from a climatic station located in the experimental basin Jizerka, on the top of the Jizerka Mountains in Czech Republic. He employed the False neighborhood method and Lyapunov Exponent test to search for chaos in the precipitation and temperature series respectively. False neighborhood technique reported degrees of freedom in precipitation and temperature as 9 and 5, respectively. Lyapunov Exponent also provided the support for the presence of chaos in the temperature series but in the case of precipitation time series the same interpretation could not be made with Lyapunov Exponent due to high intermittance nature of rainfall data.

Elshorbagy *et al.,* (2002) carried out further research investigation of chaos in daily stream flows. The Correlation dimension, the Lyapunov exponent method, the Kolomogrov method and Surrogate method are employed for the analysis of 10,000 observations. Results presented attractor with a dimension of 2.40. They configured two models using chaotic dynamics to compute the missing values of the stream flow. Using

the attractor dimension, they decided the number of parameters to be used in artificial neural network and K-nearest neighbor technique. ANN has showed the superiority in the estimation of missing stream flows than K-neighbor technique and it is attributed to the capability of the ANN's to capture the non linear dynamics and generalize the structure of the attractor on the whole data set.

Chapter 3

THEORY OF CHAOS

3.1 Introduction

French mathematical physicist Henri Poincare's, work in the late 19th century is considered as beginning of chaos theory. Lorenz, a meteorologist, is the person later to Poincare, who showed the existence of chaotic motion in the strange attractor experimentally in 1961. With the advent of computers, development has occurred in a less span of time in this field However, chaos theory is still in its infancy. Chaos theory involves more mathematics and it can be treated as part of non-linear dynamics We can apply this in various fields as a new technique, in hydrology also.

This new science of non-linear dynamics stimulated interest in many fields e.g. meteorology, economics, physiology, molecular physics, and astronomy. In hydrology, Rodriguez-Iturbe *et al.*, (1989) showed the existence of chaos in the storm rainfall data, and as a forward step Jayawardena and Lai predicted the future values of hydrologic variables with moderate accuracy. To investigate for chaos, it is necessary to understand its concepts and characterization. In view of this, a detailed discussion is presented in this chapter. The following section describes the chaotic behavior and its characterization.

3.2 What is chaos?

"*Chaos*" the word itself creates a sense of confusion, which means utter confusion and disorder. And chaos theory is a study of a system or group of connected things looking apparently erratic, complex, and almost random those are very sensitive to initial conditions, Infact, the system is deterministic

3.2.1 How it looks?

Chaotic behavior, when looked at casually, looks erratic and almost random – almost like the behavior of a system with many, many degrees of freedom, each doing it's own bit. Infact, the system is an order determined, in some sense by the equations describing the system. Now we have to explore this determinism, which distinguishes system's behavior from apparent randomness.

If we see a system with complex random like behavior, we might try to explain that behavior by either an argument based on the notion of "noise" or an argument based on "complexity." According to the noise argument the complex behavior is due to uncontrolled outside factors through which system behavior appears random. From the complexity argument, real systems are made of billions and billions of atoms and molecules. Since we cannot control precisely the behavior of all the atoms and molecules, it will lead to fluctuations and randomness in the overall behavior of the system. Technically we could say that these complex systems have many degrees of freedom and it is the activity of these many degrees of freedom that leads to the apparently random behavior. Of course in many cases, both noise and complexity might be contributing factors. The importance of chaos is that it provides the alternative explanation for the

apparent randomness stating that it depends on neither noise nor complexity. Chaotic behavior shows up in a system that is essentially free of noise and is relatively simple-only a few degrees of freedom are active. Chaos theory provides us with the tools to carry out this analysis. Small change in the parameter values produces drastic changes in some of the non-linear systems, which could be characterized as chaos. The following example can explain.

Consider the logistic map equation

$$x_{n+1} = A * x_n * (x_n - 1) \tag{3.1}$$

Where A is constant; x_n = current value; x_{n+1} = future value

The following table lists the orbits for three trajectories, each with A=3.99. One trajectory Starts from $x = 0.4000$, a second from $x = 0.401$, and a third from $x = 0\ 4005$.

Table 3.1 Trajectories for the Logistic map with A=3.99

n	x_n	x_n	x_n
0	0.4000	0.4010	0 4005
1	0.9576	0.9584	0.9580
2	0.1620	0.1591	0 1605
3	0.5417	0.5338	0 5377
4	0.9906	0.9929	0.9918
5	0.0373	0.0280	0.0324
6	0.1432	0.1085	0.1250
7	0.4894	0.3860	0.4365
8	0.9971	0.9456	0 9814
9	0.0117	0.2052	0.0727
10	0.0462	0.6507	0.2691
11	0.1758	0 9069	0.7847
12	0.5781	0 3368	0.6740
13	0.9731	0.8912	0 8767
14	0.1043	0.3870	0 4314
15	0.3727	0.9465	0.9787

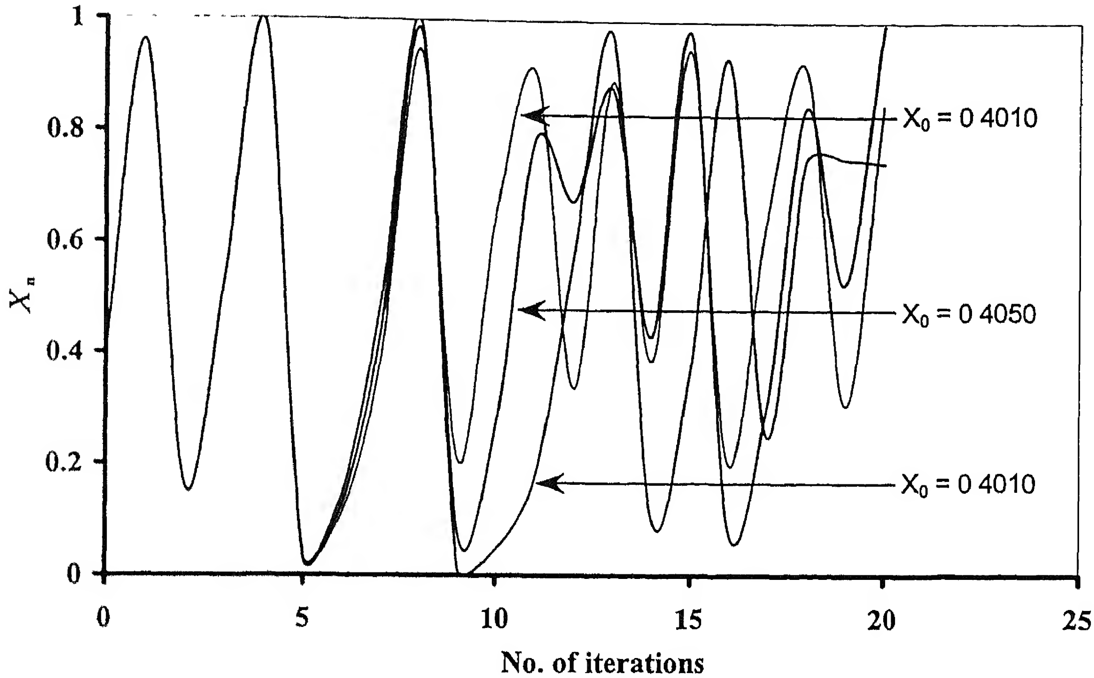


Figure 3.1: Trajectories for Logic map describing chaotic behavior

The above trajectories with a slight change in initial condition take completely different paths. This can be clearly observed after 10 iterations with a separation of 0.6 between the first two trajectories, which is known as divergence of nearby trajectories. And if we see the complete path taken by the trajectories, it looks random and creates a sense of disorder. But unlike to outside appearance these are well defined by a simple equation with a small change in initial condition. We can say that "chaos" is defined as "when looked at casually, looks erratic and almost random – almost like the behavior of a system with many, many degrees of freedom, each doing it's own bit. Infact, the system is an order determined, in some sense by the equations describing the system." Hence, we can consider the divergence of nearby trajectories in characterization of chaos. The divergence of nearby trajectories is possible for only

particular values of constant "A", implying chaos are possible only in those systems which are sensitive to initial conditions. Reducing the initial difference to half *i e* from 0.001 to 0.0005 does not mean that number iterations needed will be twice for the trajectories to get the same distance apart, creating impression that chaos involves nonlinear dynamics. Infact, it takes only one additional iteration to diverge.

3.2.2 Is long term prediction possible?

As mentioned earlier, these nearby trajectories follow definite order and generally chaotic systems can be represented by the equations, but we cannot represent the solution in the closed form of equations. Because the closed form does not represent the divergence of nearby trajectories, in which small changes in initial conditions lead to completely different paths, which is characterization of chaotic systems. To represent chaotic systems, we must integrate the equations step by step to find the future behavior. The divergence of nearby trajectories means that any small error in specifying the initial conditions will be magnified as we integrate the equations. In real systems, there is always some imprecision in specifying initial conditions Thus a small change in initial conditions leads to grossly different long-term behavior of the system so that we cannot in practice predict that long-term behavior in detail.

3.2.3 Characterization of Chaos

Chaotic behavior is characterized by divergence of nearby trajectories in the state space as a function of time, which is the separation between two nearby trajectories increasing exponentially, at least for short times. The last restriction is necessary because we are concerned with system whose trajectories stay within some bounded region of state space. There are three requirements for chaotic behavior in such a situation.

1) No intersection between trajectories.

2) Bounded trajectories

3) Exponential divergence of nearby trajectories.

These conditions cannot be satisfied simultaneously in one or two-dimensional state space.

Chaotic behavior can be described either qualitatively or quantitatively. Bifurcation diagrams and the divergence of trajectories are the qualitative representation. We want some quantitative test for chaotic behavior to distinguish chaotic nature from noisy behavior due to random, external influences. Secondly, we would like to have some quantitative measure of the degree of chaoticity. So we can see how chaotic behavior changes with changes in system parameters.

In characterizing the chaos quantitatively, we will make use of two different, but related types of description. The first type emphasizes on the dynamics of chaotic behavior. These quantifiers are the Lyapunov Exponent and Kolmogorov Entropy, which tell us how the system evolves and what happen to nearby trajectories as time goes on. The second type gives the information of geometric nature of the chaos, such as Correaltion Exponents and Fractal Dimension. If we allow the system to evolve for a long time and examine it subsequently, it may reveal the geometry of the resulting trajectories.

3.3 Phase Space Representation of Time Series:

It is not obvious that a set of sampled values of just one variable should be sufficient to capture the features of the system. Infact, if the sampling is carried out at the

appropriate time intervals and if the sequence is used cleverly, then we can indeed "reconstruct" the essential features of the dynamics in state space.

The phase space diagram can reveal the deterministic dynamics in a dynamic system. A method of reconstructing a phase space from a Univariate time series has been proposed by Packard *et al.* (1980), and put on a firm mathematical basis by Takens (1981). The dynamics of a time series $\{x_1, x_2, \ldots\ldots\ldots\ldots x_n\}$ are fully captured or embedded in the m-dimensional phase space defined by $Y_t = \{x_t, x_{t+\tau}, x_{t+2\tau}, \ldots\ldots\ldots x_{t+(m-1)\tau}\}$. The values of m should be greater than *d*, where *d* is the dimension of the attractor and τ is the delay time. According to the embedding theorem of Takens (1981), a *d*- dimensional attractor can be embedded into a (2*d*+1)-dimensional phase space to evaluate the characteristics of the dynamic system.

The delay time τ needs to be appropriately chosen. If the value of the τ is less than appropriate value then the data values will not be independent resulting in a loss of information and characteristics on the attractor structure. If τ is too large, i.e. much larger than the information decay time, then there is no dynamic correlation between the state vectors, thus causing a loss of information on the original system. The choice of τ is usually made with the help of the autocorrelation function or mutual information content (Jayawardena and Lai, 1994).

For practical applications, it is convenient to use the autocorrelation function of the time series $\{x_1, x_2, \ldots\ldots\ldots\ldots x_n\}$. The delay time τ may be chosen as the lag time at which the autocorrelation falls below a threshold value which is commonly

defined as $\frac{1}{e}$, especially if the autocorrelation function is approximately exponential (Tsonis and Elsner 1988). Another method is to take the time lag that first generates a zero autocorrelation if the autocorrelation function crosses the zero line (Mpitsos *et al.*, 1987).

Many natural systems like rainfall and stream flow are characterized by attractors, but the system does not converge with time to a point nor to a cyclic trajectory. Thus process never exactly repeats itself. This type of attractor is very peculiar one: It is low dimensional, being contained in a reduced portion of the low dimensional phase space (e.g. rectangle, a sphere, a box, or a hyper sphere), never crosses itself and contains every possible frequency in a broadband spectrum. This type of attractor is known as strange attractor and the Correlation Integral can find the dimension of the attractor.

3.4 Correlation Exponent:

We can construct any dimensional phase space portrait with above-mentioned method for the given time series or data set. In the phase space portrait, construct a circle of radius 'r' centered about any arbitrary point of the time series or data set. Count the number of points N(r) falling inside the circle of radius 'r'. Normalizing the count N(r) gives the Correlation Integral of the process C(r).

$$C(r) = \lim_{n \to \infty} \frac{1}{N^2} \{\text{number of pairs } i, j \text{ whose distance} \left| x_i - x_j \right| < r \}$$

Where i, j are indices ordering the points along a trajectory containing a total of 'N' points.

3.4.1 Calculation of Correlation Exponent

Grossberger and procaccia defined the Correlation integral as follows:

$$C(r) = \lim_{N\to\infty} \frac{2}{N(N-1)} \sum_{\substack{ij \\ (1\le i<j\le N)}} H\left(r - \left|Y_i - Y_j\right|\right) \tag{3.2}$$

Where

H = Heaviside step function with H (u) =1 for u>0 and H (u) = 0 for u <= 0

$$u = r - \left|Y_i - Y_j\right| \tag{3.3}$$

r = radius of sphere centered on Y_i or Y_j and

N= number of data points.

The norm $\left|Y_i - Y_j\right|$ may be any of the usual norms the maximum norm, the diamond norm, or the standard euclidian norm. If any attractor lies in the given dynamical system, then Correlation Exponent varies with radius as follows:

$$C(r) \cong \alpha\, r^{\upsilon} \tag{3.4}$$

Where α is constant; v is known as Correlation Exponent or Correlation Exponent of the attractor. It can be computed from the slope of log C(r) Vs log (r) line drawn for the different embedding dimensions and procedure to calculate it as follows:

First we compute the Correlation function, C(r), for various embedding dimensions of the given time series or data set. Then we plot the graph of log C(r) Vs log(r) for various embedding dimensions. Then we calculate the slope of the linear portion of the graph, in which the above mentioned power law is valid and this linear region is

known as scaling region. The determination of the scaling region is rather subjective, since in many cases the log C(r) Vs log (r) curve is not a straight line over a very large range of radius 'r'. The scaling region depends upon the radius interval and data size. The slope can be calculated from a least-squares fit of a straight line to the scaling region data. This slope is known as the Correlation Exponent of the attractor.

3.4.2 How Deterministic Process and Stochastic Process differ?

The process, which comes from deterministic dimension, will have limited number of degrees of freedom that may capture the main features of the dynamics. Thus, if we construct higher and higher dimension phase space, a point will be reached where the dimension equals the number of degrees of freedom and increasing the dimension of the representation will not affect the relation N(r) ~ r^{ν} for an infinite data set. In other words, v will remain constant after a certain dimension of the phase space and high dimensional representations will be redundant in regards to the information they contain. This property can be used in search of the chaotic dynamics (Rodriguez-Iturbe *et al.,* 1989). In contrast, Correlation Exponent continuously increases without any bound with embedding dimension for a stochastic process.

3.4.3 Phase Space Dimension:

The Correlation Exponent of the attractor provides information on the dimension of the phase space to reconstruct the attractor and on the number of variable for the evaluation of the given dynamic process. According to the embedded theorem of Takens (1981), to characterize a dynamic system with an attractor dimension d, an ($m=2d+1$)-dimensional phase space is adequate, whereas Abarbanel *et al.* (1991) suggested that, in practice, m>d would be sufficient. According to the Fraedrich (1986), the nearest

integer above the Correlation Exponent of the attractor provides the minimum dimension of the phase space essential to embed the attractor, while the value of the embedding dimension at which the saturation of the correlation exponent occurs provides an upper bound on the dimension of the phase-space sufficient to describe the motion of the attractor (Sivakumar *et al.*, 1999).

3.4.3 Limitations of Correlation Exponent:

The following factors influence the calculation of the Correlation Exponent, which can be considered limitations in the computation of Correlation Exponent

3.4.3.1 Finite Data

According to some authors (Smith 1988; Nerenberg and Essex 1990), the number of data points required for a reliable dimension estimate increases exponentially with the embedding dimension used for the phase space reconstruction. Numerous attempts have been and are being made to provide some guidelines on this issue (e.g. Smith, 1988; Havstad and Ehlers, 1989; Nerenberg and Essex, 1990; Ramsey and Yuan, 1990). Smith suggested 42^m data points for the m-dimensional attractor time series. Nerenberg and Essex (1990) flawed the Smith's procedure and suggested the minimum number of data points as $10.^{2+0.4m}$ Ramsey and Yuan (1990) concluded that for small sample sizes, dimension could be estimated with upward bias for chaotic systems and with downward bias for random noise as the embedding dimension is increased. They proved that, due to these bias effects, a Correlation Exponent estimate of 0.214 could imply an actual Correlation Exponent value of as high as 1.68 (Sivakumar, 2000). Rodriguez-Iturbe *et al.* (1989) suggested the continuation of decreasing the sample size until significant changes in results are observed to obtain the minimum number of data points. Proparto and Rudolfi

(1996) computed the Correlation Exponent by varying the number of data points and got the same results. Thus none of the studies addressing the issue of data size has been able to provide a clear-cut guideline on the minimum data size for the Correlation Exponent estimation. But it can be understood that large data set which is able to represent the dynamic changes in a time series is sufficient to compute reliable Correlation Exponent. And the inclusion of large number of points (or vectors) on the reconstructed phase space increases the scaling region, which makes the calculation of Correlation Exponent easy. In contrast, few data points make the slope determination difficult. Therefore, it may be necessary to have a large data size for Correlation Exponent estimation.

3.4.3.2 Effects of Noise:

The Correlation Exponent calculation can also be affected by the presence of noise-either real noise in experimental data or round-off noise in a numerical computation. If the average "size" of the noise is R_n, then noise will dominate the structure of the attractor for which $R<R_n$. Since the noise is supposedly random, the noise-dominated data will tend to be spread out uniformly in the state space, and for small values of 'r', the dimension of the attractor and the dimension of the phase space will be the same. The presence of noise influences the estimation of Correlation Exponent primarily from the identification of the scaling region. Noise may corrupt the scaling behavior at all length scales, but its effects are significant at small length scales. It has been observed that even small levels of noise significantly complicate the estimation of Correlation Exponent, a quantity that, in principle, should be straight forward to measure. Although a wide variety of nonlinear noise reduction methods have been made available in the literature over the past decade (e.g. Schriber and Grassberger, 1991; Schreiber, 1993; Grassbeger *et al.*,

1993), their applicability to hydrological data has been tested only recently (Proporato and Ridolfi, 1997; Sivakumar *et al.*, 1999c). The failure of the majority of the studies to address the problem of noise and it's possible effects on chaos identification in hydrological data forms another side of criticism of the validity of such studies.

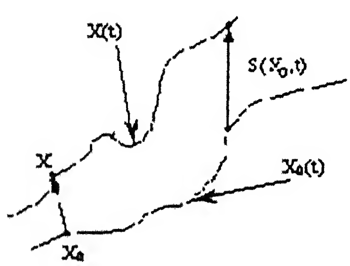
3.4.3.3 Data with Gaps:

The given data set sometimes will have two distinct ranges, which produce the gaps in the data set/values. This will result different scaling regions in the log C(r) vs. log (r) graph. If the Correlation Exponent is different the standard Correlation Exponent procedure yields an average Correlation Exponent.

3.5 Lyapunov Exponent

Lyapunov Exponent is another method to quantify the chaos. Lyapunov Exponent is a measure of the rate of attraction or repulsion from a fixed point in space. So, we could apply this notion to the divergence of nearby trajectories at any point in state space. And, as divergence of trajectories is considered as characterization of chaotic behavior, we can use this parameter to quantify the chaos

3.5.1 Calculation of Lyapunov Exponent:



Consider two points in state space, let x_0 be the one initial point and x a nearby initial point. Let x_0(t) be the trajectory that arises from that initial point, while *x(t)*

is the trajectory that arises from an other initial point. The time development equation is assumed to be

$$\dot{x}(t) = f(x) \tag{3.5}$$

Since we assume that x is close to x_0, we can use a Taylor series expansion to write

$$f(x) = f(x_0) + \left.\frac{df(x)}{dx}\right|_{x_0} (x - x_0) + \frac{\partial^2 f(x)}{\partial x^2}.(x - x_0)^2 + \dots\dots\dots\dots\dots \tag{3.6}$$

Since the trajectories represent equations of the dynamic system, the separation 's', between the trajectories will be function of time. And sensitive dependence can arise only in some portions of a system, so this separation is considered as function of the location the initial value. And it can be written as

$$s(x_0, t) = x - x_0 \tag{3.7}$$

The rate of change of distance between the two trajectories given by

$$\left.\begin{aligned} \dot{s} &= \dot{x} - \dot{x}_0 \\ \dot{s} &= f(x) - f(x_0) \\ \dot{s} &= \left.\frac{df}{dx}\right|_{x_0} (x - x_0) \end{aligned}\right\} \tag{3.8}$$

(neglecting all derivatives of order higher than first order terms)

The solution to which is,

$$s(x_0, t) = s(x_0, t = 0)\, e^{\lambda t} \tag{3.9}$$

This can also written as

$$s_1 = s_0\, e^{\lambda t} \tag{3.10}$$

Differentiating the above equation with respect to time, we find

$$\dot{s} = \lambda\, s(t=0)\, e^{\lambda t} \tag{3.11}$$

$$\dot{s} = \lambda\, s \tag{3.12}$$

Comparing equations (3.8) and (3.12) yields

$$\lambda = \left.\frac{df(x)}{dx}\right|_{x_0} \tag{3.13}$$

Thus we see that if λ is positive, then the two trajectories will diverge. Trajectories will converge if λ is negative and will attract to a stable fixed point or stable periodic orbit. Negative Lyapunov Exponents are characteristic of dissipitative or non-conservative systems. Such systems exhibit asymptotic stability. The more negative the exponent, the greater the stability. Super stable fixed points and super stable periodic points have a Lypaunov Exponent of $\lambda = -\infty$. This is something akin to a critically damped oscillator in the system, which heads towards its equilibrium point as quickly as possible.

A Lyapunov Exponent of zero indicates that the system is in some sort of steady state mode. A physical system with this exponent is conservative. Such systems exhibit Lyapunov stability. This can be observed in the phase portrait of the two identical simple harmonic oscillators with different amplitudes. Because the frequency is independent of the amplitude, the phase space portrait of the two oscillators would be a pair of concentric circles.

In the chaotic systems, the divergence of trajectories will occur for a short period of time, this is the reason for showing the positive value of λ. Generally, all real

systems are dissipative systems in which trajectories approach the attractor. So "λ" will be negative. So we define a chaotic system to be a system, which has at least one positive average of Lyapunov Exponent.

3.5.2 Wolf (1985) Algorithm:

Thus from the above discussion, it can be observed that Lyapunov Exponent is a function of the separation between the trajectories. In multi-dimensional phase space, when separation between trajectories (s_1) becomes too large departing from exponential behavior, then we shift to new nearby trajectory and define a new separation value $s_0(t)$. In this way, the calculation is averaged over different regions of the phase space, and the Lyapunov Exponent is estimated from the following expression

$$\lambda_1 = \frac{1}{t_M - t_0} \sum_{k=1}^{M} \log_2 \frac{L'(t_k)}{L(t_{k-1})} \tag{3.14}$$

Where M is the total number of replacement steps; $L(t_{k-1})$ is the Euclidian distance between the point $\{x(t_{k-1}), x(t_{k-1-\tau}), \ldots\ldots x(t_{k-1-(m-1)\tau})\}$ and it's nearest neighbor, and $L'(t_k)$ is the evolved length of $L(t_{j-1})$ at a time t_k. The above expression was proposed by wolf *et al.*, (1985). Figure 3.2 gives schematic representation to calculate Lyapunov Exponent and the procedure to calculate it is as follows:

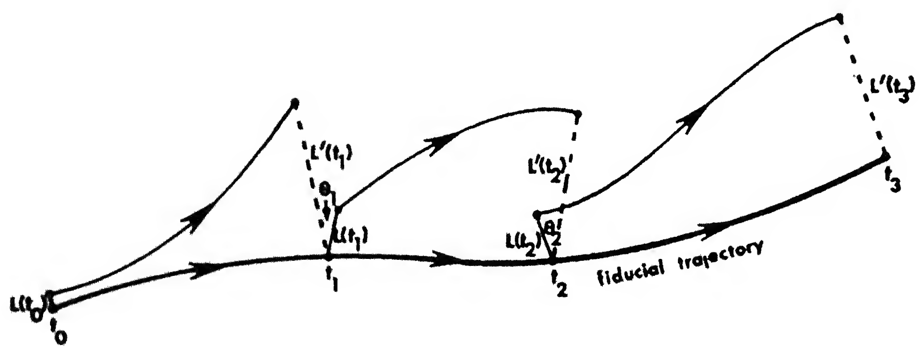


Figure 3.2 A Schematic representation to estimate Lyapunov Exponent

For the given the time series $x(t)$, an m-dimensional phase portrait is reconstructed with delay coordinates. Then locate the nearest neighbor in the phase space by calculating the Euclidian distance from the initial point $\{x(t_0),.........,x(t_0+[m-1]\tau\}$ to the coordinates of the constructed phase space. The separation between the two trajectories is denoted as $L(t_0)$ at first initial point. After a small time t_1, the initial separation length becomes $L'(t_1)$. The length element is propagated through the attractor for a time short enough time (called evolution time) so that only small scale attractor structure is likely to be examined. If the evolution time is too large we may see L' shrink as the two trajectories, which define it, pass through a folding region of the attractor. It would lead to an under estimation of the attractor. Now, we look for a new data point that satisfies the following two criteria reasonably well: (a) it's separation, $L'(t_1)$ from the evolved fiducial point should be small, and, (b) the angular separation between the evolved and replacement element should be small. If an adequate replacement point cannot be found, then retain the points that were being used. This procedure is repeated until the fiducial trajectory (It is considered as reference trajectory to find divergence and angular separation for the evolved points) has traversed the entire data file and Lyapunov Exponent can be estimated using equation 3.14.

It will require the phase space dimension, evolution time and delay time as input parameters to calculate the Lyapunov Exponent for the given time series. The values of the input parameters should be chosen carefully, because these input parameters influence the calculation of Lyapunov Exponent. The influencing parameters of the Lyapunov Exponent are described as follows:

3.5.3 Selection of the Input Parameters for Lyapunov Exponent:

Embedding Dimension: While reconstructing the attractor using delay coordinates we embed the attractor for any sufficiently large value 'm', but for accurate estimation of the Lyapunov Exponent, it's value should be chosen carefully. If the attractor is reconstructed in the low dimensional phase space, then folding of the attractor will result. As a result largely separated elements on the original attractor will cluster and become replacement elements. Such elements are liable to grow in the reconstructed attractor for a short period, making enormous contribution to the estimated exponent.

If 'm' is chosen too large then noise presented in the data decreases the density of the elements on the attractor. It makes hard to find replacement points in the estimation of the exponent. So increasing 'm' from a minimum required value has the effect of unnecessarily increasing the level of contamination of the data. So it is advisable to check the stationarity of the results with 'm' to ensure robust estimates.

Delay time: We embed the attractor for any delay time like for any dimensional phase space while constructing the attractor using delay coordinates. But for accurate estimation we have to choose the value carefully. The value of the delay time should neither be so small that the attractor stretches out along the line $x = y = z$, nor should it be so large that $m\tau$ is much larger than orbital period.

Evolution Time: Value of the evolution time between replacements should be carefully chosen as it can influence both orientation error and frequency of replacement steps.

Maximizing the propagation time of volume elements is highly desirable as it both reduces the frequency with which orientation errors are made and reduces the calculation considerably because element propagation involves much less computation than element replacement. Results of noise-free model systems showed that too frequent replacements cause loss of phase space orientation and too infrequent replacements allow volume elements to grow overly large and exhibit folding.

Chapter 4

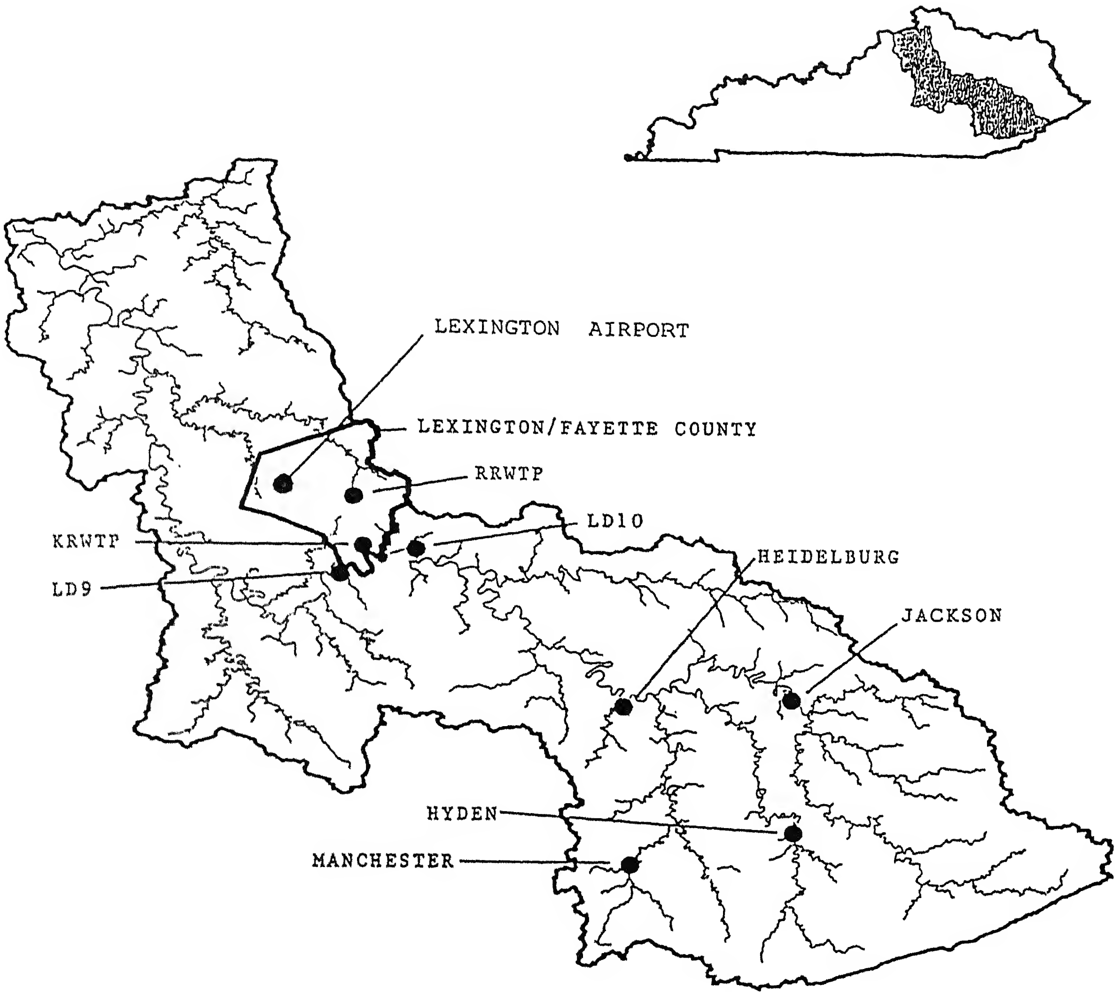
INVESTIGATION OF CHAOS

4.1 Introduction:

This chapter describes the data employed and the methodologies used to investigate the existence of chaos in the hydrologic variables. Algorithms, calculation of the parameters employed to quantify the chaos, and the problems involved in the calculation are described earlier. To investigate for chaos, we employed autocorrelation function, Correlation Exponent, and Lyapunov Exponent on different data sets with different time scales. Among these techniques autocorrelation plot, which is drawn between autocorrelation value and lag time, provides valuable information on delay time to be used in phase-space reconstruction. Correlation Exponent and Lyapunov Exponent give the details about chaos, if exists.

4.2 Data Employed

The investigation of chaos was carried out on two different data sets observed at the Kentucky River in the state of Kentucky, USA. A map of the Kentucky River basin, with its rainfall and flow stations, is shown in Figure 4.1. The Kentucky River has three forks in its upstream reaches, namely the north, middle, and south forks. The first data set is rainfall observations of 10 - years record length from North Fork Kentucky River basin. The second data set includes both rainfall and runoff observations for a period of 30-years, from the whole Kentucky River Basin. The drainage area of the basin is 10244 km^2.



LD 10 = Lock and Dam 10

KRWTP = Kentucky River Water Treatment Plant

RRWTP = Richmond Road Water Treatment Plant

Figure 4.1 Kentucky River Basin

The daily rainfall data of 10-years period at London situated in NFKY River Basin were investigated for the existence of chaos. Later, the investigation was extended to whole Kentucky River Basin, which includes upstream and downstream observations of both rainfall and runoff. Second data set contains five different rainfall observations, namely, Jackson, Hyden, Manchester, Heidelburg, and Ford Lock 10 and two different runoff observations, namely, Heidelburg and Lock 10. Among the above stations Jackson, Hyden, Manchester, and Heidelburg are located in the upstream of the Kentucky River Basin. Ford Lock 10 and Lock 10 are the station of down stream of the Kentucky River Basin. In this way second data set represents the whole basin with a large record length unlike first data set.

In addition, the hydrologic variables have been analyzed for temporal and spatial variation from a chaotic perspective. Temporal analysis is carried out on both data sets, while spatial variation is examined on the second data set only. Daily rainfall at London from NFKY river basin is transformed into weekly rainfall by simply adding successive 7-daily values. In the same way the flow at Lock 10 is transformed into 5-days and 7-days runoff to analyze the temporal variation. To understand the effects of spatial variation, the averaged daily rainfall observations from three rainfall stations, Jackson, Hyden and Manchester were investigated. And similar analysis is performed on the averaged runoff data also. In this way rainfall and runoff processes of the whole Kentucky River Basin is analyzed from temporal and spatial aspects.

4.3 METHODOLOGIES

4.3.1 Autocorrelation Function:

A computer program was written to calculate auto correlation function for each of the data set. The autocorrelation function can be calculated using the following equation:

$$r(k)=\frac{\sum_{i=1}^{n-k}X_i.X_{i+k}-\frac{\sum_{i=1}^{n-k}X_i\sum_{i=1}^{n-k}X_{i+k}}{(n-k)}}{\left[\sum_{i=1}^{n-k}X^2{}_i-\frac{\left(\sum_{i=1}^{n-k}X_i\right)^2}{n-k}\right]^{\frac{1}{2}}\left[\sum_{i=1}^{n-k}X^2{}_{i+k}-\frac{\left(\sum_{i=1}^{n-k}X_{i+k}\right)^2}{n-k}\right]^{\frac{1}{2}}} \qquad (4.1)$$

Where $r\ (k)$ is the auto correlation function value for lag k

'k' is the lag or number of time intervals between the observations being considered;

'n' is the number of observations;

'X' is the variable of observation;

Maximum lag time was generally taken as one-third of the number of data elements for the calculation of the auto correlation function value. Thus we can calculate auto correlation function values for corresponding lag time which starts from zero lag time to maximum lag time from the program. Auto correlation function value denotes the linear dependence among the single variable separated in time by the lag time.

After calculating the autocorrelation function, we draw the auto correlation plot (called Correlogram) taking auto correlation values on y-axis and lag time on x-axis. We can extract the data features by observing the correlogram. Of course, it is not necessary that our interpretation always may be correct.

4.3.2 Correlation Exponent:

Most of the studies have used Correlation Exponent to investigate for the existence of chaos in the hydrological processes. Comparatively, with other parameters, it will provide more information, about chaos in the hydrologic processes, and at the same time it has complexity in the calculation process. Theoretically, it is influenced by data size, noise in the data, and gaps in the data, which were addressed previously. But the practical problems are quite different, which will be explained below.

To calculate the Correlation Exponent, first we need to have correlation integral values for different embedding dimensions for a given data set. Values of the Correlation integral were obtained by a computer program written in 'C'. Later calculation i.e. calculation of Correlation Exponent involves manual work. The following flow chart describes the steps involved for the Calculation of the Correlation exponent for a given data set. The problems faced in the calculation of the correlation exponent are briefly addressed in the subsequent sections.

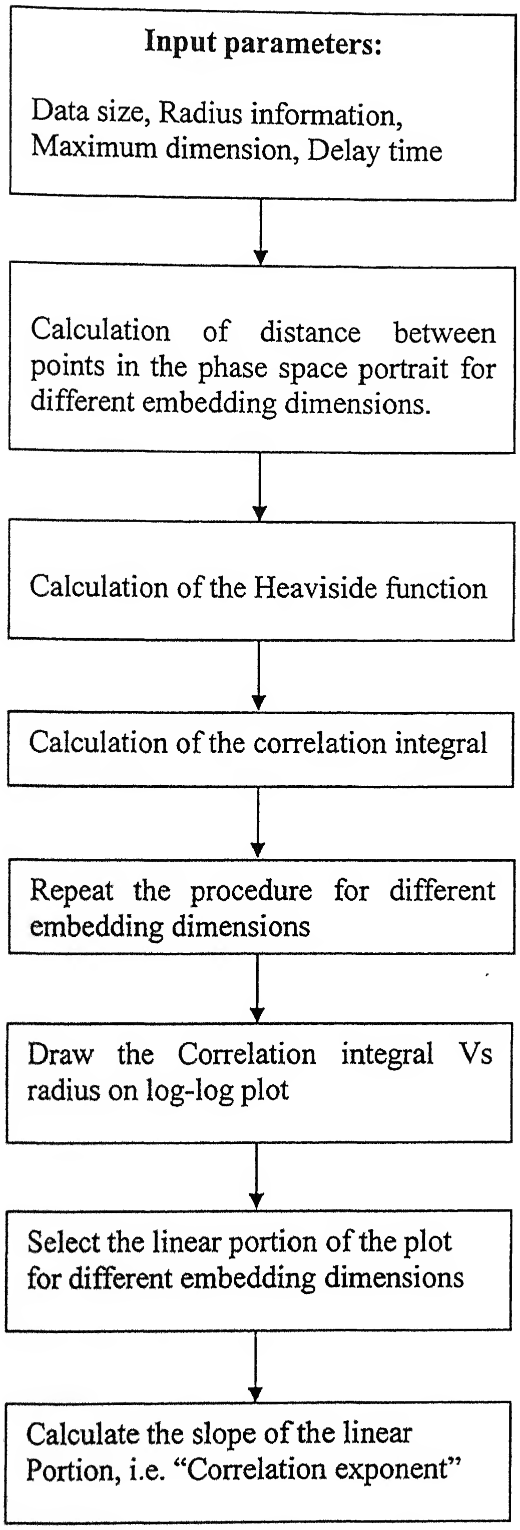


Figure 4.2 Flow Chart for Calculation of Correlation Exponent

After calculation of the correlation Integral, we plot the values on the log-log graph. From these plots, we estimate the Correlation exponent. Unlike other calculations that can be done by programming, Calculation of Correlation exponent needs more manual work, and requires experience. The procedure is discussed here briefly.

Draw the log-log plot by taking the correlation integral values on y-axis and radius values on the x-axis for different embedding dimensions. The crucial part of the calculation of the Correlation exponent is the selection of the scaling region of the graph for particular embedding dimension. The reason behind the selection of the scaling region is the assumption that the number of points falling in a particular radius follows exponential distribution. On the log-log graph exponential distribution converts into linear relationship. The selection of the scaling region is rather subjective. This depends on the data size and radius interval. And it varies with every embedding dimension. So it is necessary to watch carefully each line of the corresponding embedding dimension in the graph to select scaling region. As there are no special guidelines in the selection of the linear portion, one will become efficient through practice only.

With experience, observing the graph keenly can also point to the existence of the chaos in the physical process. According to theory, saturation of the Correlation Exponent can be treated as the existence of chaos. Correlation Exponent is nothing but slope of the linear portion of the graph, and if the lines of the correlation integral Vs radius become parallel in the linear portion then it would mean that the correlation exponent has saturated. In other words, when the slope of the scaling region,

with increasing embedding dimension becomes saturated or constant, then it would indicate that the existence of chaos in the time series of the physical process.

Before employing this method, it is good to have a glance at the data. Knowing the minimum and maximum values will guide the selection of the radius interval and maximum radius size. For the one-dimensional calculation, maximum radius is simply the maximum distance between the points. With increasing embedding dimension it will vary. The selection of the radius interval also plays a vital role. The radius interval has to change according to the order of the magnitude of the physical variable under investigation e.g. rainfall will have smaller values comparatively to runoff values. So when working with rainfall data the radius interval should be small unlike to runoff data. Otherwise the selection of linear portion is misleading, through which entire calculation can be erroneous. In this study, in the calculation of the Correlation function, C(r), radius, r, values considered on the $\log_2$ scale and on the $\log_{10}$ scale for rainfall and runoff observations respectively. Sometimes, it is possible to have two or more linear portions with different slopes in the Correlation integral Vs radius plot. Then Correlation exponent will be obtained by averaging the slopes of different linear portions. If the value of the Correlation Exponent does not saturate then it would mean that the process could be considered as a stochastic process.

4.3.3 Lyapunov Exponent Method:

This section describes the influencing parameters of the Lyapunov Exponent other than those discussed in the previous chapter. The selection of the variables and the working of the program are explained in the following sections.

In the process of the calculation of the Lyapunov Exponent, we will replace the co-ordinates by the other co-ordinates selectively. Selection will be done based on two criteria: one is based on the maximum distance consideration and second is based on the minimum distance consideration. In the computer code, these can be represented by SCALMX and SCALMN respectively. SCALMX is the maximum distance allowed in replacing the co-ordinates and SCALMN is the minimum distance needed to replace the co-ordinates. Maximum distance to replace the point will maintain the orientation of the trajectory and minimum distance will not allow the noise effects as the distance is confined. There is no guideline to give the values for these variables. Based on the nature of the data one can decide the values.

Another element to be considered is 'angular variation' while replacing the elements. Minimum and Maximum angular variations are represented in the code by THMIN and ANGLMX respectively. It will also maintain the orientation of trajectory through the minimum and maximum angular variations.

For a particular dimension and evolution time it will search replacement point satisfying the linear separation and angular separation criteria. It will

search for the new trajectory till it reaches end of the trajectory. In this way, we can calculate the Lyapunov Exponent for different parameters by varying evolution time and embedding dimension.

However, the above-mentioned parameters will not influence the calculation of Lyapunov Exponent in a certain range, and as there is no guideline for any parameter, it is better to test the stationarity of the Lyapunov Exponent for various values of the parameters. So in the output we can have Lyapunov Exponent for different embedding dimensions, for different evolution time, and for different linear separation values. Such type of output data easily reveals the range of the parameters for which stationarity of the Lyapunov Exponent prevails.

This chapter discussed the methodology followed in the present work and quantifiers of the chaos were described in detail. With this methodology described in this chapter, the results, and analysis of results are discussed in the next chapter.

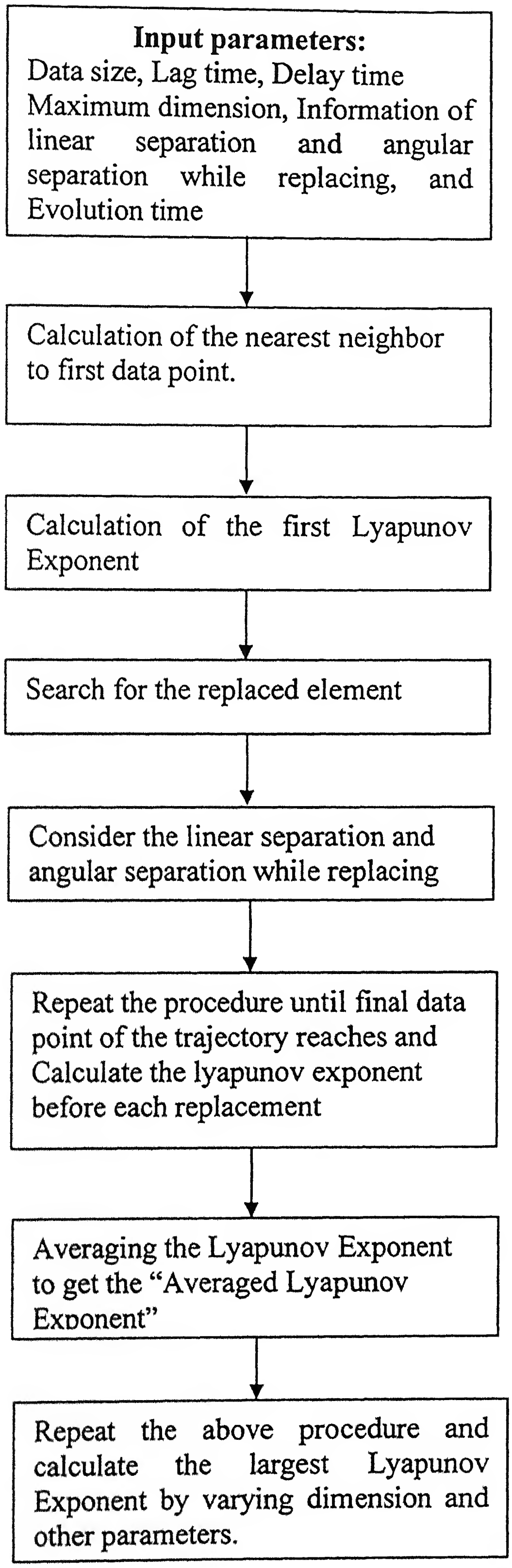


Figure 4.3 Flow Chart for Calculation for Lyapunov Exponent

Chapter 5

RESULTS AND DISCUSSIONS

In this chapter, results in terms of both Correlation Integral and Lyapunov Exponent on different data sets are presented and discussed. Each data set is analyzed based upon the Correlation Integral plots and Lyapunov Exponent plots. In the first part of the analysis, Correlation Integral plots were developed for each data set. Then Correlation Exponent was calculated for each embedding dimension and plot of these results provide the dimension of the attractor. The second part of the analysis includes the selection of Lyapunov Exponent based on Lyapunov Exponent plots and tables. These plots were obtained by plotting Lyapuov Exponent as a function of evolution time for each embedding dimension. The following observations have been made from the Lyapunov Exponent plots: 1) Width of the band in which Lyapunov Exponent fluctuates decreases with increasing embedding dimension, and 2) Either increasing or decreasing variation of Lyapunov Exponent observed with the respective evolution time. However, almost similar values of Lyapunov Exponent were observed over a particular range of evolution time for each embedding dimension. This phenomenon is known as consistency or stationarity of the Lyapunov Exponent. Over this consistency range, maximum Lyapunov Exponent and corresponding evolution time were tabulated for each embedding dimension. From this table of values, minimum, maximum, and average values of the Lyapunov Exponent were calculated and tabulated.

The results are discussed in the following order. First, we discuss the results of daily rainfall data of the London, North Fork Kentucky River Basin. Secondly

we discuss the results of daily rainfall and flow values of the whole Kentucky River Basin at both upstream and downstream stations. And in the subsequent sections, results associated with the temporal and spatial variations of the both rainfall and flow data are analyzed. Figure 5.X.1 shows the relationship between the correlation function, C(r), and the radius, r, for various values of embedding dimension, *m*, and similarly Figure 5.X.2 shows the relationship between the correlation exponent, ν, and embedding dimension, *m*, for different record lengths of the hydrological variables. (Please note that X value varies from 1 to 17.)

5.1 NFKY River Basin Results

Correlation Exponent and Lyapunov Exponent values of the NFKY River basin are presented in the Table 5.1 and Table 5.2, respectively graphical results of the same location are presented in the Figures 5.1.1 and 5.1.2. As can be observed from Figure 5.1.2, the correlation exponent values do not seem to saturate, indicating existence of stochasticity or 'no chaos' in the data. Based on Porporato and Rudolfi (1996), there may be evidence of existence of dynamics in daily rainfall data at London, Kentucky with correlation exponent (ν) =3.87 at embedding dimension (m) = 20. However, we need more than one quantifier such as Lyapunov Exponent to conclude about the existence of chaos.

From the consistency interval of the Lyapunov Exponent, maximum, minimum and average Lyapunov Exponent were calculated for daily rainfall data at London and are presented in the Table 5.2. As can be noticed, the Lyapunov Exponent are

Table 5. 1 Results of Rainfall from Correlation Integral Analysis

Data Set	Correlation Exponent (ν)	Embedding Dimension (m)
Daily Rainfall at		
London	3.87	20
Jackson	3.98	20
Hyden	4.25	20
Manchester	4.13	20
Heidelburg	4.03	20
Ford lock10	3.83	20
Average Daily Rainfall	5.03	20

Table 5. 2 Lyapunaov Exponents for Daily Rainfall at Kentucky River Basin

Data Set	LE_{min}	LE_{max}	LE_{avg}	No. Of LEs in the Consistency range (%)	Chaos Exists
Daily Rainfall at					
London	0.001087	0.002281	0.00155	60	YES
Jackson	0.001307	0.002423	0.001722	44	YES
Hyden	0.001210	0.001979	0.001511	50	YES
Manchester	0.001453	0.002595	0.002028	60	YES
Heidelburg	0.001612	0.002688	0.001950	50	YES
Ford lock 10	0.001658	0.002312	0.001899	40	YES
Average Daily Rainfall	0.000984	0.002141	0.001412	60	YES

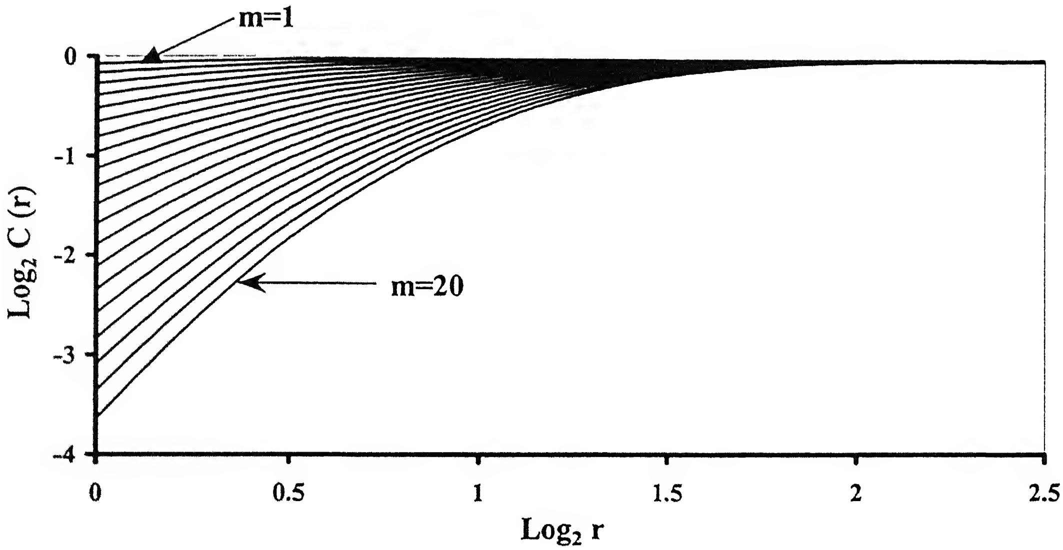


Figure 5.1.1: Correlation Integral plot of Daily Rainfall at London

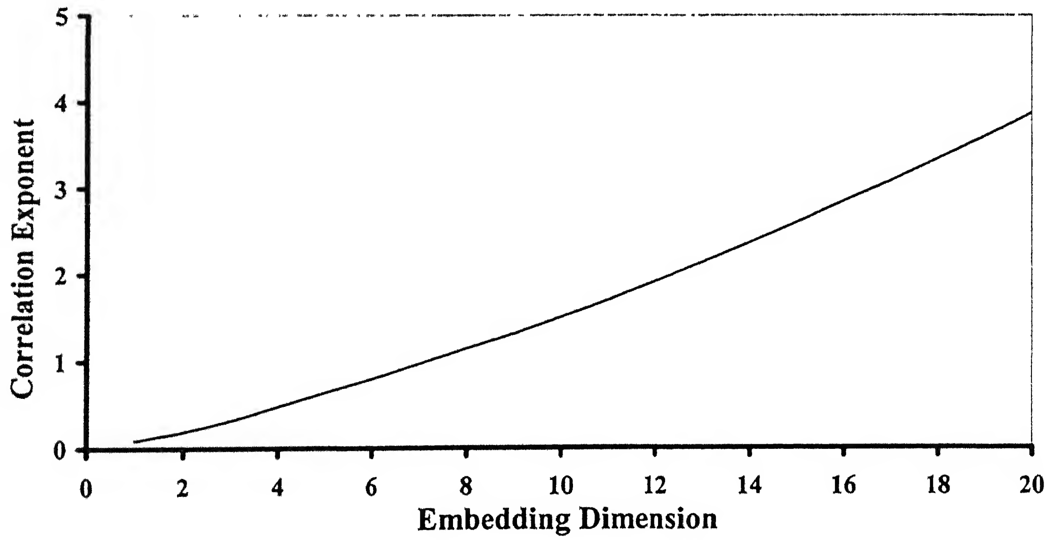


Figure 5.1.2: Correlation Exponent plot for Daily Rainfall at London

positive Lyapunov Exponent is sufficient for the existence of chaos". In addition to the above positive values, many other positive Lyapunov Exponents values were obtained for this dataset.

Hence, Lyapunov Exponent method is conclusive as far as the existence of the chaos in daily rainfall data at London, Kentucky is concerned, and existence of low-dimensional dynamics in daily rainfall data at London, Kentucky cannot be ruled out according to Porporato and Ridolfi (1996).

5.2 Kentucky River Basin Results

5.2.1 Results for Rainfall data

The daily rainfall data from five stations i.e. Jackson, Hyden, Manchester, Heidelburg, and Ford Lock 10 of Kentucky River Basin were analyzed using Correlation Integral and Lyapunov Exponent methods for the existence of chaos. Results of the above data series, in terms of Correlation Exponent and Lyapunov Exponent, are presented in Table 5.1 and in Table 5.2, respectively graphical results of the above data series are presented from Figure 5.2.X to Figure 5.6.X (Please note that value of 'X' varies from 1 to 2). Considering the daily rainfall at Jackson, from the Figure 5.2.2, the Correlation Exponent value was found to be 3.98 for embedding dimension 20 as per Proporato and Ridolfi (1996). From Table 5.2, it can be observed that maximum, minimum and average values of the Lyapunov Exponent are positive for this dataset. In addition to these positive values, we found many other positive Lyapunov Exponents.

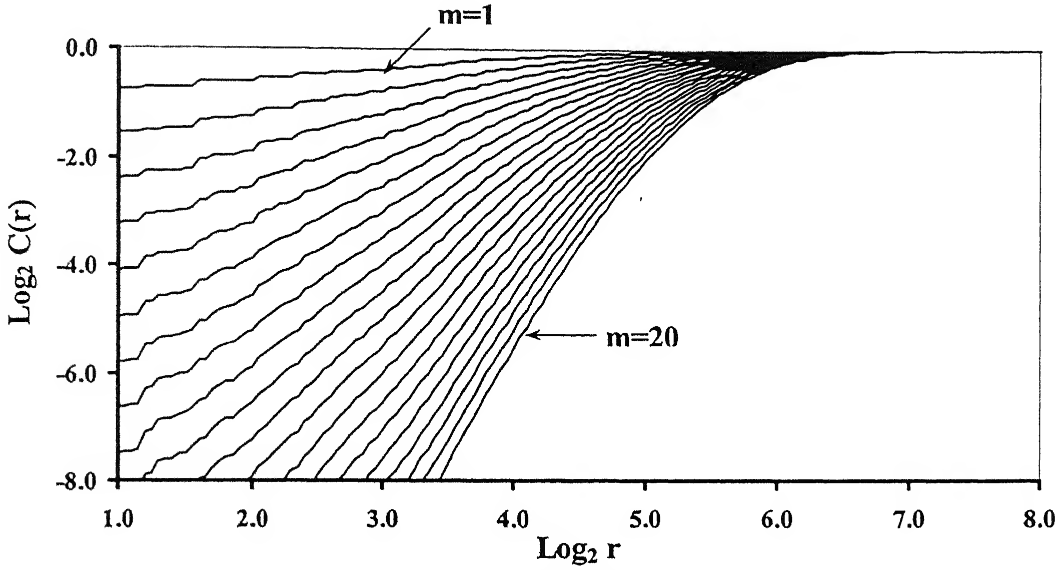


Figure 5.2.1: Correlation Integral plot of Daily Rainafll at Jackson

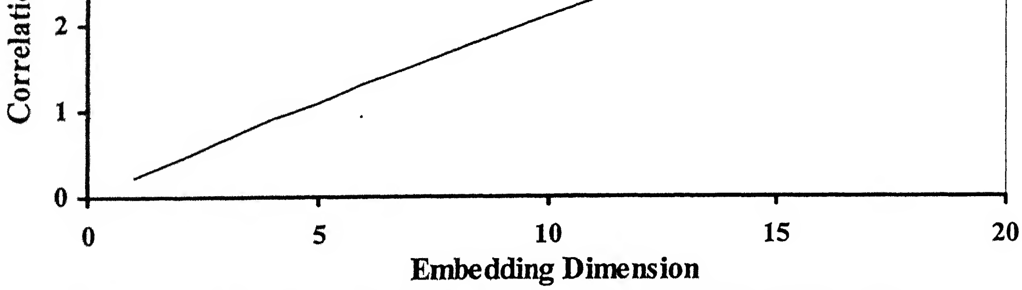


Figure 5.2.2: Correlation Exponent plot for Daily Rainfall at Jackson

results of daily rainfall data at Jackson, Kenucky, and existence of low dimensional dynamics cannot be ruled out as per Porporato and Ridolfi (1996). Similar analyses were carried out for Hyden, Manchester, Heidelburg and Ford Lock 10. The daily rainfall data of stations have shown similar results. The results of Correlation Integral analysis and Lyapunov Exponent analysis are presented in Table 5.1 and Table 5.2, respectively.

Correlation Exponent of the data series of Kentucky River Basin i.e. daily rainfall at London, Jackson, Hyden, Manchester, Heidelburg, and Ford Lock 10 does not show saturation with embedding dimension unlike Jaiwardena and Lai (1994), and Sivakumat *et al.,* (1998) for the daily rainfall data of different record lengths. Saturation of the Correlation Exponent with embedding dimension was clearly observed for the daily rainfall of 11 years duration at Hong Kong and different record lengths of daily rainfall at Singapore with low dimensional attractor, supporting the existence of the chaos (for e.g. Jaiwardena and Lai., 1994. and Sivakumat *et al.,* 1998.). Saturation of the Correlation Exponent provides a lot of information regarding the influencing variable of the process and minimum embedding dimensions to embed the attractor. However according to Porporato and Rudolfi (1996) "Even though a real plateau in the correlation exponent plot does not exist and there is no complete saturation, such a behavior of the Correlation Integral allows the possibility that a low dimensional dynamics might be present in the phenomenon." Based upon the above assumption Correlation Exponents at embedding dimension (m)=20 were estimated for the above data series. The above estimated values can be treated as clues for the existence of chaos in the daily rainfall, because value of Correlation Exponent in the current study are less than those obtained by Porporato and

Ridolfi (1996). However, from the above results it can be said that the number of dimension sufficient to embed the attractor is surely more than 20 in the above data sets.

Correlation Exponent of a time series represents the variability or irregularity of the values in the series. A series with a higher variability in values will have a higher Correlation Exponent, which, in turn, indicates higher complexity in the dynamics of the process. The Correlation Exponent values for daily rainfall at Hong Kong and Singapore are about 1.00. Correlation Exponent values for daily rainfall in KRB ranges from 3.83 to 4.25. Thus it can be said that daily rainfall data of KRB has higher complexity in its dynamics as compared to daily rainfall at Hong Kong and Singapore. Decrease in the value of the Correlation Exponent of the same duration of the data series was observed while moving towards downstream of the KRB. Maximum value of the Correlation Exponent i.e. 4.25 observed for daily rainfall at Hyden, upstream location among the five locations of KRB, and minimum Correlation Exponent i.e. 3.83 observed for daily rainfall at Ford Lock 10, downstream reaches of the KRB. The variation in the Correlation Exponent may be due to variation of the hydrological and meteorological conditions.

For deterministic systems, the presence of positive Lyapunov Exponent is taken to be the definition of chaos. Thus to check chaos, it is sufficient to consider largest (maximum) Lyapunov Exponent. Lyapunov Exponents were estimated using Wolf *et al.*, (1985) algorithm and selected from the consistency range are presented in the Table 5.2. Even though these values are very low, although positive. The largest Lyapunov Exponent determines the average horizon of predictivity for the system. However, in order for the largest Lyapunov Exponent being used to estimate the limit of prdictibilty, it must be

computed for a specific value of the embedding dimension of the phase-space and evolution time. This can be accomplished by a technique called 'false nearest neighbor' method (e.g. Stehlik, 2001). However, the method adopted in the present study (Wolf *et al.,* 1985) for the estimation of the Largest Lyapunov Exponent is not desirable to use for finding the limit or prediction, as this is not calculated for a specific values of the embedding dimension and evolution time. We adopted this method to check whether the largest Lyapunov Exponent is positive or not. The values of the Lyapunov Exponent are ranges from 0.0001979 bits/day to 0.002688 bits/day. Overall consistency range of Lyapunov Exponent was observed for 50% of evolution time range.

5.2.2 Results for Flow Data

Daily average flow at two stations, namely, Heidelburg, and Ford Lock 10 at Kentucky River have been considered. Heidelburg is upstream of Ford lock 10. The results of both data are discussed below. Results of the Correlation Exponent and Lyapunov Exponent are presented in Table 5.3 and Table 5.4, respectively graphic results of the Correlation Exponent are presented in Figures 5.7.2 and 5.8.2 for the above flow series, respectively.

5.2.2.1 Flow at Heidelburg

Figure 5.7.2 shows the relationship between Correlation Exponent and the embedding dimension values for the average daily flow at Heidelburg. Figure 5.7.2 also shows that the Correlation Exponent increases with an increase in the embedding dimension up to a certain point, and saturates beyond that dimension. Unlike to daily

Table 5. 3 Scaling Results of Correlation Integral Analysis:

Data Set	LE $_{min}$	LE $_{max}$	LE $_{avg}$	No. Of LE_s in consistency range (%)	Chaos Exists
Average daily flow at Heidelburg	0.001378	0.001743	0.001538	50	YES
Ford Lock 10	0.001788	0.003301	0.00214	50	YES
5-day stream flow at Ford Lock 10	0.001661	0.002736	0.002069	100	YES
7-day stream flow at Ford Lock 10	0.00285	0.00407	0.003437	92	YES

Table 5.4 Scaling Results for Lyapunov Exponent at London

Data Set	LE $_{min}$	LE $_{max}$	LE $_{avg}$	No. Of LEs in consistency range (%)	Chaos Exists
Daily Rainfall at London	0.001087	0.002281	0.00155	60	YES
2-day Rainfall at London	0.000421	0.000963	0.000633	80	YES
5-day rainfall at London	0.000289	0.000682	0.00437	100	YES
7-day Rainfall at London	0.000308	0.000607	0.00412	100	YES

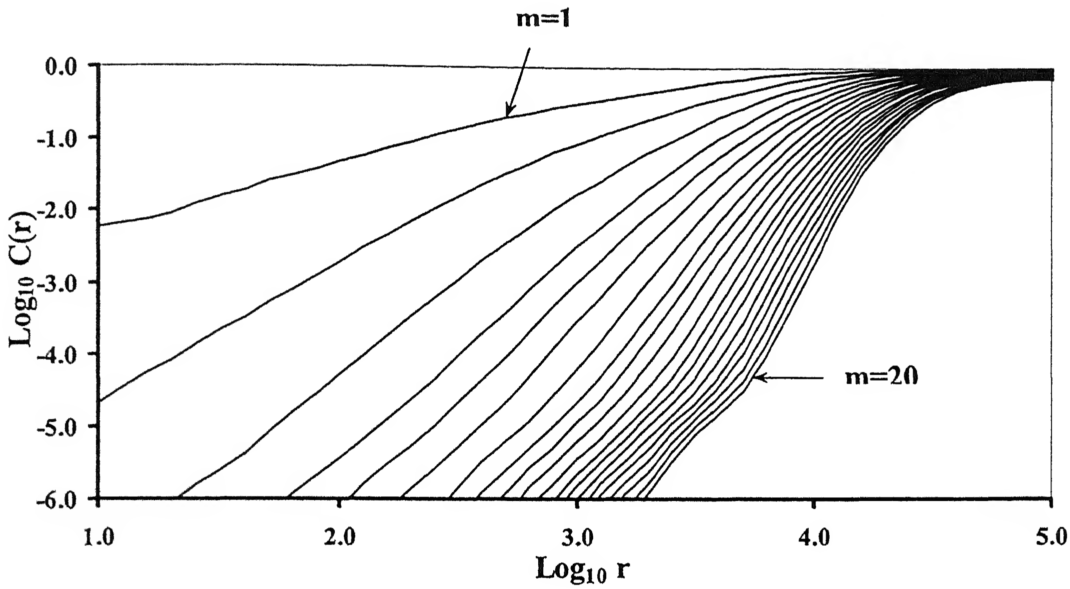


Figure 5.7.1: Correlation Integral plot of Average Daily flow at Heidelburg

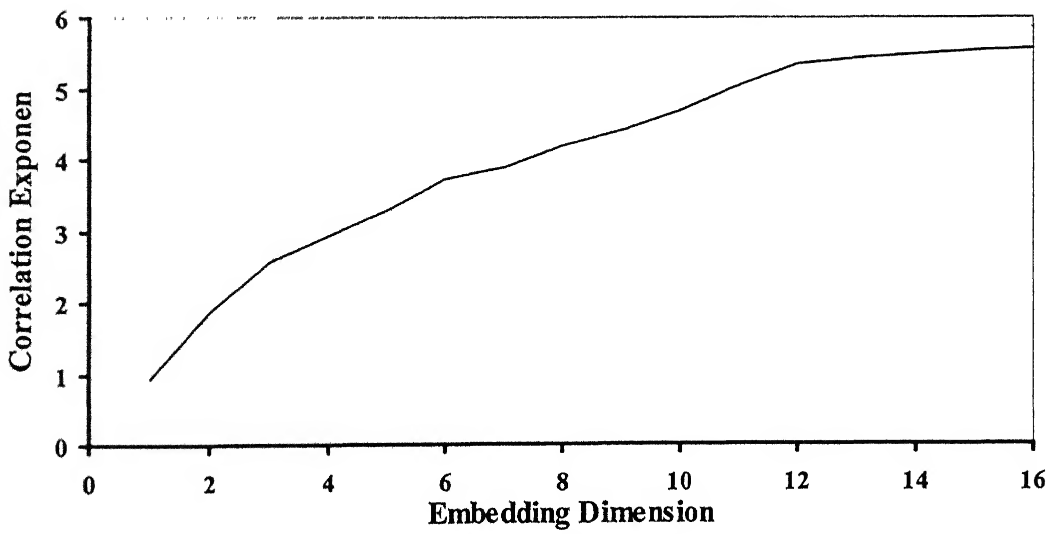


Figure 5.7.2: Correlation Exponent plot for Average Daily flow at Heidelburg

rainfall data, Correlation Exponent plot showed saturation of the Correlation Exponent clearly for the average daily runoff at Heidelburg. The saturation value of the Correlation Exponent is approximately 5.42 at the embedding dimension (m) = 13. Therefore the minimum dimension of the phase space essential to embed the attractor of the runoff process is '6' (i.e. nearest integer above the saturation value of correlation exponent) while the dimension of the phase space sufficient to embed the attractor is 13 (the embedding dimension at which saturation occurs).

Table 5.4 presents the values of the maximum, minimum and average Lyapunov Exponents for this data set. All of these were found to be positive supporting the existence of the chaos.

5.2.2.2 Flow at Ford Lock 10

Figure 5.8.2 shows the relationship between Correlation Exponent and the embedding dimension values for the average daily flow at Ford Lock 10. Similar to average daily flow at Heidelburg, Correlation Exponent plot (Figure 5.8.2) showed saturation of the Correlation Exponent clearly for the average daily runoff at For Lock 10. The saturation value of the Correlation Exponent is approximately 3.97 at the embedding dimension (m) = 10. Therefore the minimum dimension of the phase space essential to embed the attractor of the runoff process is '4' (i.e. nearest integer above the saturation value of correlation exponent) while the dimension of the phase space sufficient to embed the attractor is 10 (the embedding dimension at which saturation occurs). From Table 5.4, it can be observed that maximum, minimum and average values of the Lyapunov Exponents are positive for this dataset. In addition to these positive values, we found

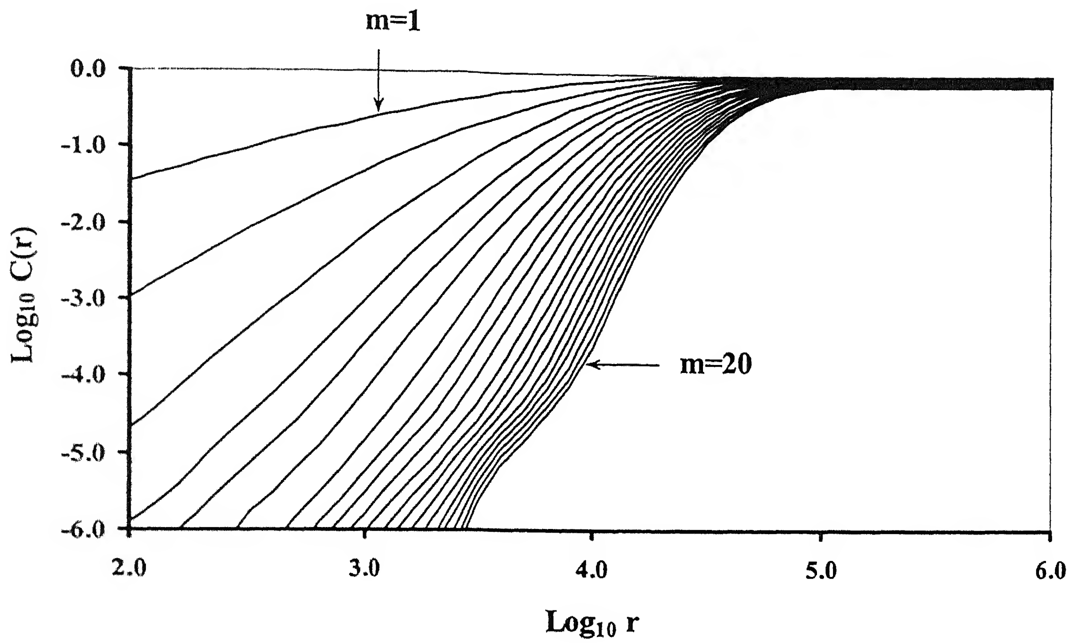


Figure 5.8.1: Correlation Integral plot of Average Daily flow at Lock10

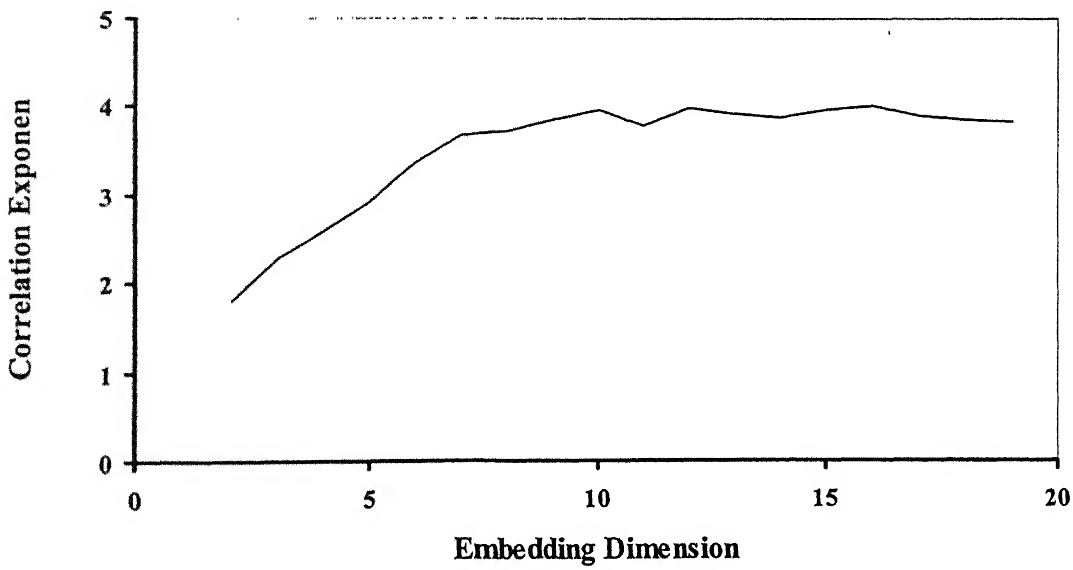


Figure 5.8.2: Correlation Exponent plot for Average Daily flow at Lock10

many other positive Lyapunov Exponent estimates that strengthen evidence for the chaotic dynamics, in average daily flow data at the Lock 10 of Kentucky River Basin.

Results of the Correlation Integral and Lyapunov Exponent for the average daily flow series of the Kentucky River provide more informative than the results of the daily rainfall series of KRB. Saturation of the Correlation Exponent gives the information about the controlling variables of the process whereas results of the daily rainfall provide only clues for the existence of the chaos. Reasons for showing saturation of Correlation Exponent in the average daily flows might be less noisy nature and less spatial variation of the flow data unlike to complex phenomenon of rainfall process. Correlation Exponent of the average daily flow at Ford Lock 10 is lower than the Correlation Exponent of the average daily flow at Heidelburg. It suggests that the flow at Ford Lock 10 has low variability indicating that the dynamics of the processes are less complex relatively flow at Heidelburg.

Positive Lyapunov Exponents of the above data series supports the existence of the chaos in the average daily flows. It can observe that the data having low variability (flow at ford Lock 10) has higher Lyapunov Exponent and low Correlation Exponent. Also the data having higher variability data (flow at Heidelburg) has higher Correlation Exponent and low Lyapunov Exponent. It suggests that an inverse relationship exists between Correlation Exponent and Lyapunov Exponent at least based on the results obtained in the current study. Both data sets showed 50% consistency range for the selection of the maximum Lyapunov Exponents.

S. Boringdon and F. Lisi (1999) analyzed the mean daily discharge of record length 11 years of Adige River, in northeast Italy, and found the Correlation Dimension and largest Lyapunov Exponent as 2.8 and 0.007, which are in the range of values that we have obtained for the daily runoff of KRB. Correlation Exponent of Adige river is less than that for the daily flows of KRB, but Lyapunov Exponent of Adige river is greater than that for the daily flows of KRB. It supports the assumption that the data has maximum Correlation Dimension will have lower Lyapunov Exponent and vice versa.

It can be said that Correlation Integral method has given strong evidence of chaos, and Lyapunov Exponent method is also conclusive as far as the existence of chaos in average daily flow at Ford Lock 10 Kentucky is concerned.

In addition, the hydrologic variables have been analyzed for temporal and spatial variation from a chaotic perspective: The results corresponding to the temporal and spatial aspects are discussed next.

5.3 Results for Scaling Effects

Scaling effects in time for both rainfall and runoff data are being analyzed in this study. Two stations, one each from NFKY and KRB, were selected for the purpose of investigating scaling effects in rainfall data. These are London, Kentucky in NFKY and Jackson, Kentucky in KRB. The time frames selected for investigating temporal scaling effects in rainfall data include one, two, five and seven days; and that for runoff data include one, five, and seven days. The flow data from Lock 10 of KRB were selected for

the investigation of temporal scales as this located most long stream in the basin. The findings of the temporal scaling effect in hydrologic variables are discussed below.

5.3.1 Scaling Effects in Rainfall data

2-day rainfall data were obtained by adding successive two values of daily rainfall. Similarly, 5–day rainfall data and 7-day rainfall were obtained by adding successive five and seven day values of daily rainfall, respectively. Investigation of the chaotic dynamics in the different resolutions of the rainfall data was carried out by employing both Correlation Integral and Lyapunov Exponent algorithms.

5.3.1.1 NFKY River Basin Results

Table 5.5 and Table 5.6 show Correlation Exponent estimates and Lyapunov Exponent estimates, respectively. Table 5.5 also includes some statistical parameters of the different resolutions of the rainfall at London. Graphical results are presented from Figure 5.9.X to Figure 5.11.X.

Correlation Exponent plot of the 2-day rainfall data i.e. Figure 5.9.2 shows slightly steeper slope than that for the daily rainfall data at London. From the Figure 5.9.2, Correlation Exponent value can be estimated as 6.57 at embedding dimension 20.

Correlation Exponent plot of 5-day rainfall i.e. Figure 5.10.2 shows steeper slope than that for daily rainfall and 2-day rainfall. From the above figure Correlation Exponent can be estimated as 12.06 at embedding dimension (m) = 20 according to

Table 5. 5 Scaling Results of Correlation Integral Analysis:

Data set	**London**					**Jackson**				
Scaling	*Data points*	*% of zeros*	*Cv*	v	*m*	*Data points*	*% of zeros*	*Cv*	v	*m*
Daily rainfall	4381	63.2	2.39	3.92	20	10,958	65.6	2.49	3.98	20
2-day rainfall	2190	45.1	1.79	6.57	20	5,479	47.3	1.91	7.51	20
5-day rainfall	876	16.9	1.15	12.06	20	2,191	17.0	1.25	12.21	20
7-day rainfall	625	7.8	0.92	11.09	16	1,565	8.3	1.06	11.13	20

Table 5.6 Scaling Results for Lyapunov Exponent at London

Data Set	LE_{min}	LE_{max}	LE_{avg}	No. Of LEs in consistency range (%)	Chaos Exists
Daily Rainfall at London	0.001087	0.002281	0.00155	60	YES
2-day Rainfall at London	0.000421	0.000963	0.000633	80	YES
5-day rainfall at London	0.000289	0.000682	0.00437	100	YES
7-day Rainfall at London	0.000308	0.000607	0.00412	100	YES

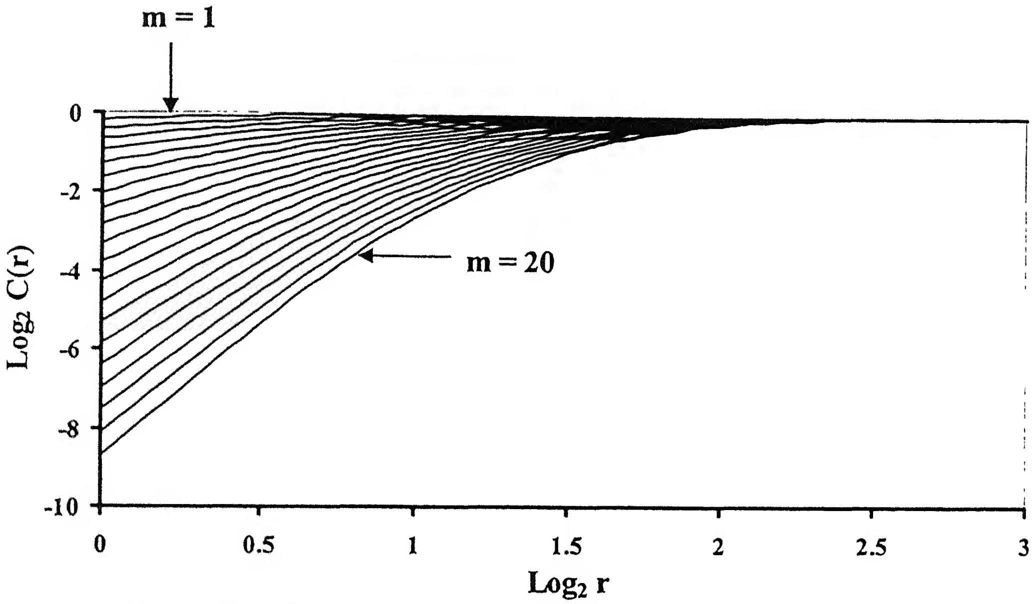


Figure 5.9.1: Correlation Integral plot of 2-day Rainfall at London

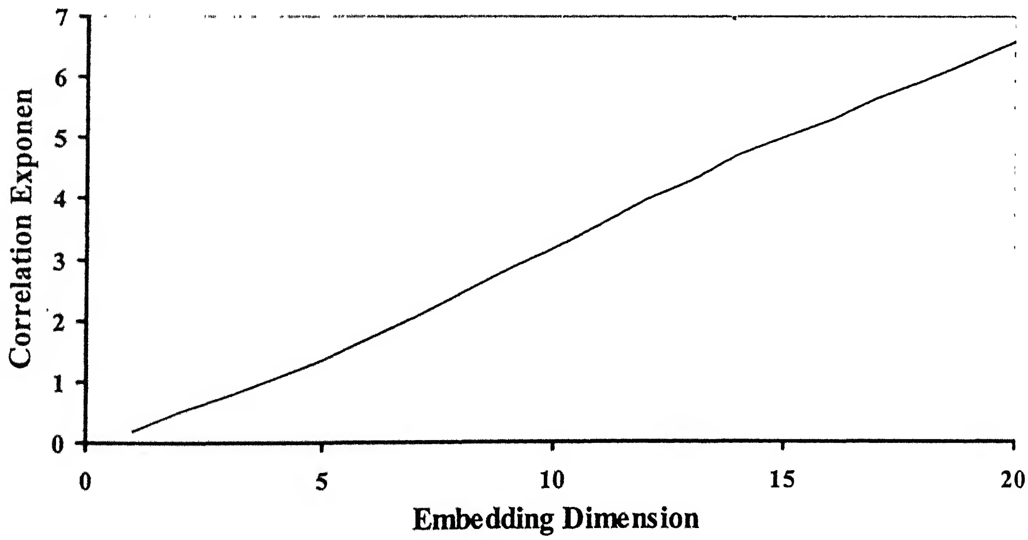


Figure 5.9.2: Correlation Exponent plot for 2-day Rainfall at London

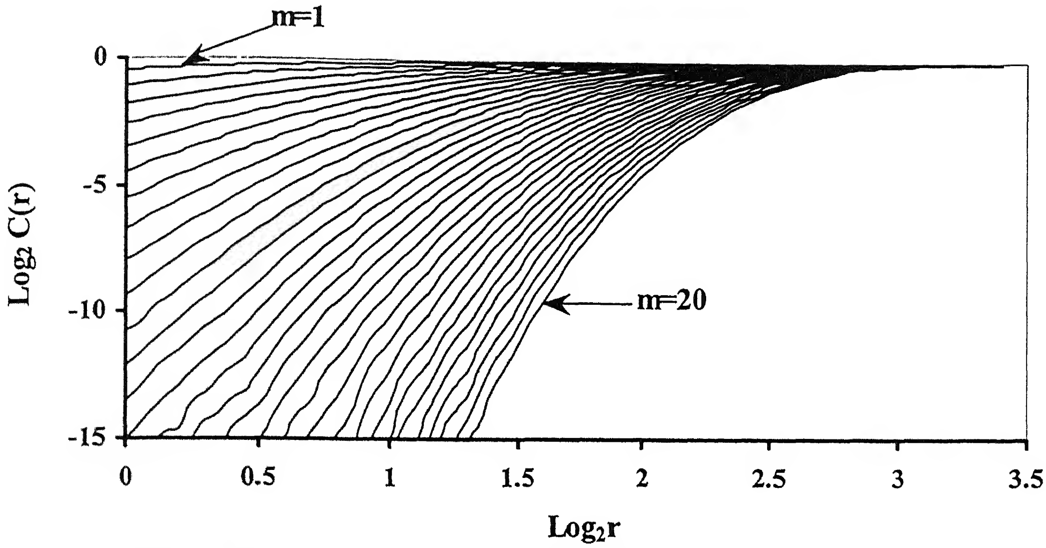


Figure 5.10.1: Correlation Integral plot of 5-dayRainfall at London

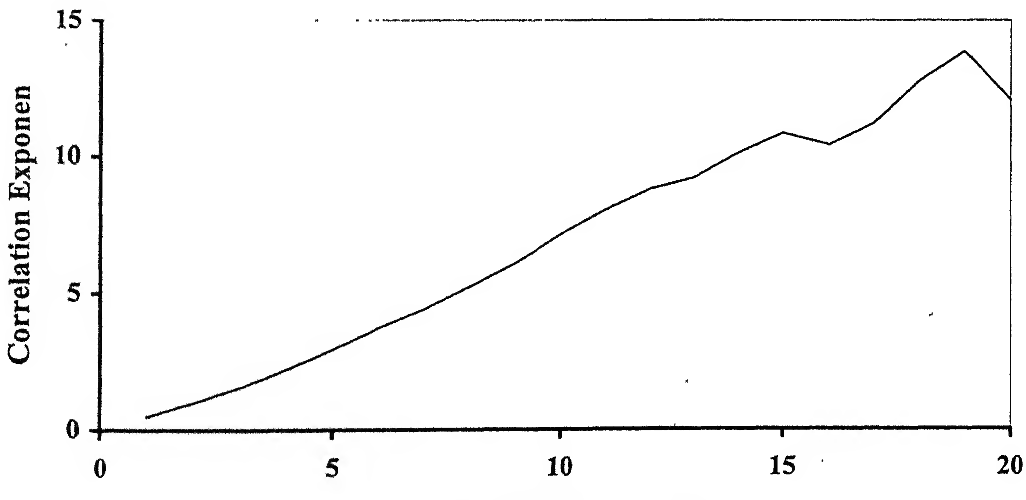


Figure 5.10.2: Correlation Exponent plot for 5-day Rainfall at London

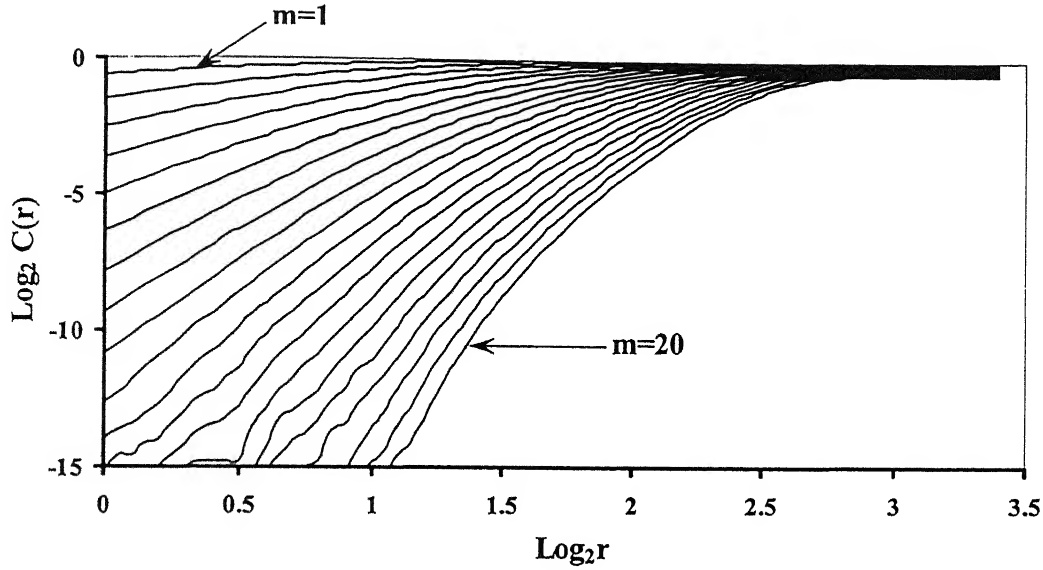


Figure 5.11.1: Correlation Integral plot of 7-day Rainfall at London

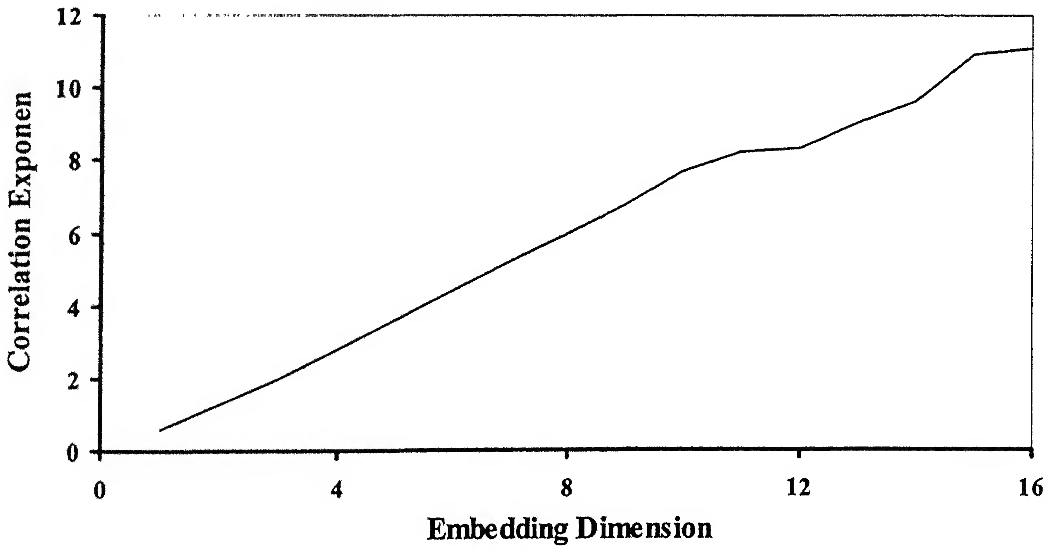


Figure 5.11.2: Correlation Exponent plot for 7-day Rainfall at London

Porporato and Ridolfi (1996). Figure 5.11.2 shows relationship between Correlation Exponent and embedding dimension for the 7-day rainfall at London. From the above figure it is observed that Correlation Exponent is increasing continuously unlike to daily rainfall data. The Correlation Exponent were calculated till embedding dimension 'm' = 17 only. After m = 17 onwards the graphs did not show linear portion. At this embedding dimension estimated Correlation Exponent is 11.09.

Maximum Lyapunov Exponent of the above data sets is presented in the Table 5.6. All these are found to be positive supporting the chaotic dynamics for all the data series i.e. daily, 2-day, 5-day and 7-day rainfall. All the above data series are also having negative values beyond the consistency range and number of negative values increase with increase in embedding dimension.

To understand the effect of scaling in rainfall data, investigation was carried out through Correlation Integral and Lyapunov Exponent methods on the different resolutions of the rainfall data at London, Kentucky. Correlation Exponent plots of the above data sets are similar to the plots obtained for daily rainfall at KRB. None of the plots of the above data sets showed saturation and it was observed that slope of the correlation exponent plot became steeper with decrease in the resolution of the rainfall data. Based upon the Porporato and Ridolfi (1996), milder slope of the plateau in the correlation exponent plot could be considered as the clue for the existence of the chaos. But there are no guidelines to consider a particular slope as the milder slope or steeper slope. Even though low-resolution data showed steeper slopes in the correlation exponent

plots relatively to the high-resolution data, we are unable to make decision about the existence of chaos. For the 7-day rainfall at London, Correlation Exponent was estimated till embedding dimension (*m*) '17' because after that scaling region was not found. The reason for that might be the reduction in the total number of data points for the long resolution data series, which may lead to the underestimation of the Correlation Exponent.

The values of the coefficient variation (*CV*), defined as the ratio of standard deviation to mean are also presented in the Table 5.5. Like Correlation Exponent, it represents the variation of the data. From Table 5.5, an increase in Correlation Exponent and decrease in the value of Coefficient of Variation was observed with decrease in the resolution of the data. High-resolution (daily rainfall) data has the maximum coefficient variation and minimum Correlation Exponent as against to the low-resolution (7-day rainfall) data, which has minimum coefficient variation and maximum Correlation Exponent (assuming the value is underestimated) among various resolutions of the data. This may be due to various reasons to which the method of Correlation Integral is very sensitive. Underestimation of the Correlation Exponent for higher resolution data or overestimation of the Correlation Exponent for lower resolution data might be the reason for the above contrarary result. The reason of which might be the presence of large number of zeros in the rainfall data. It explained below.

In the presence of a large number of any single value in a time series, the reconstructed hyper surface in phase-space will tend to a point and may result in a significant under estimation of the Correlation Exponent (Tsonis *et al.*, 1994 and

Sivakumar., 2001). Presence of large number of zeros in higher resolution data (daily rainfall data) may be the cause of the underestimation of the Correlation Exponent. According to Sivakumar *et al.,* 1999b and Sivakumar., 2000, presence of noise in the data could also result in an overestimation of the Correlation Exponent. Lower resolution data 2-day, 5-day, and 7-day have the more noise than daily rainfall, as it is obtained by successive addition of daily rainfall data, which increases cumulatively noise in lower resolution data series. This magnified error implies randomness or stochasticity.

From Table 5.6, it can be observed that the value of Lyapunov Exponent decreases with a decrease in the resolution of the data, which is similar to the behavior of the coefficient of variation. This is more on the expected lines.

It can be said that Correlation Exponent method is not robust and sensitive to noise levels and showed ambiguous results, whereas, Lyapunov Exponent method gives positive Lyapunov Exponents supporting the existence of chaos in the different resolution for the rainfall data at London, Kentucky.

5.3.1.2 Scaling Results at Jackson:

Analysis carried in this section is similar to the analysis carried in the previous section but with large number of observations. It was performed to understand the effect of Correlation Integral method on scaling effects with increase in data size of the daily rainfall of the same basin. Table 5.5 and Table 5.7 show Correlation Exponent estimates and Lyapunov Exponent results, respectively, Table 5.5 also includes some statistical parameters of the different resolutions of the rainfall at Jackson. Graphical

results are presented from Figure 5.12.X to Figure 5.14.X. (Please note that value of 'X' are 1 and 2)

Correlations Exponent plots of the different resolutions of rainfall at Jackson are similar to the Correlations Exponent plots for the different resolution of rainfall at London. Correlation Exponent plot of the 2-day rainfall data i.e. Figure 5.12.2 shows steeper slope than daily rainfall data at Jackson. From the Figure 5.12.2 we found Correlation Exponent value as 7.51 at embedding dimension 20 as per Porporato and Ridolfi, 1996. Similarly, Correlation Exponent values were calculated for 5-day and 7-day rainfall data and are presented in Table 5.5. From Table 5.7, it can be observed that maximum, minimum and average values of the Lyapunov Exponent are positive for the above datasets. In addition to these positive values, we found many other positive Lyapunov Exponents.

Scaling effects in the rainfall at Jackson showed similar behavior that was observed in the rainfall at London, i.e. Correlation Exponent value increases as the resolution decreases i.e. 2-day, 5-day and 7-day rainfall series have higher Correlation Exponent relative to the daily rainfall data at Jackson.

Minimum, Maximum and Average values of Lyapunov Exponent over a consistency range were calculated and presented in Table 5.7. These results support the existence of the chaos in 2-day, 5-day, and 7-day rainfall at Jackson, Kentucky concerned. All the above data series have less number of negative Lyapunov Exponents relative to

different resolution of rainfall data at London. After embedding dimension 'm' = 20 considerable increment observed in the number of negative Lyapunov Exponent estimates.

Thus, it can be said that Correlation Exponent method is not robust and insensitive to noise levels and showed ambiguous results, whereas, Lyapunov Exponent method gives positive Lyapunov Exponents supporting the existence of chaos.

Table 5. 8 Scaling Results for Lyapunov Exponent at Jackson:

Data Set	LE $_{min}$	LE $_{max}$	LE avg	No. Of LE s in consistency range (%)	Presence of chaos Remarks
Daily Rainfall at Jackson	0.001307	0.002423	0.001722	44	YES
2-day Rainfall at Jackson	0.000407	0.000776	0.000567	100	YES
5-day rainfall at Jackson	0.000281	0.000482	0.000357	100	YES
7-day Rainfall at Jackson	0.000167	0.000339	0.000236	100	YES

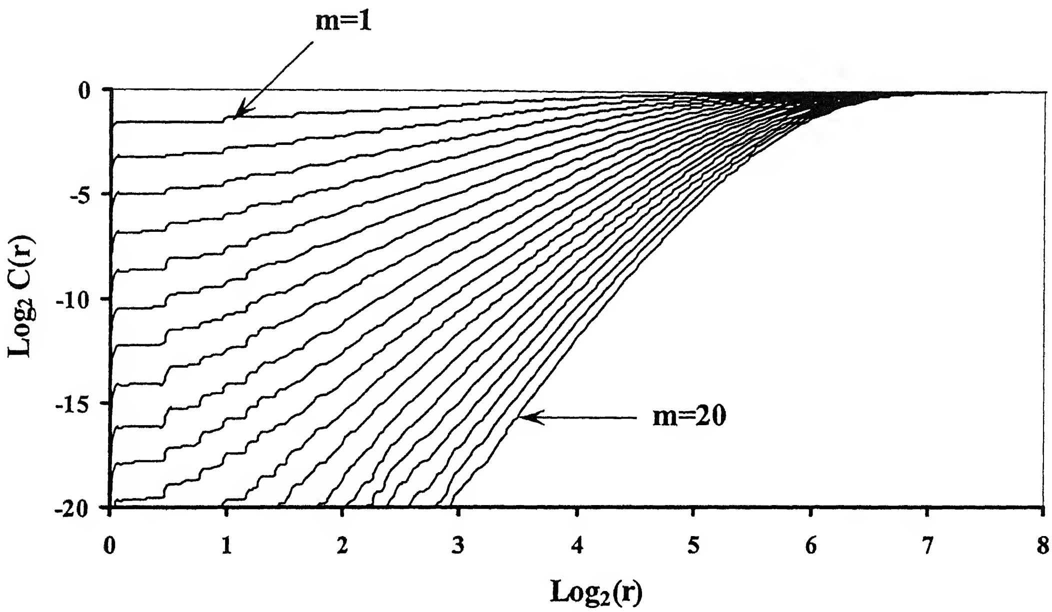


Figure 5.12.1: Correlation Integral plot of 2-day rainfall at Jackson

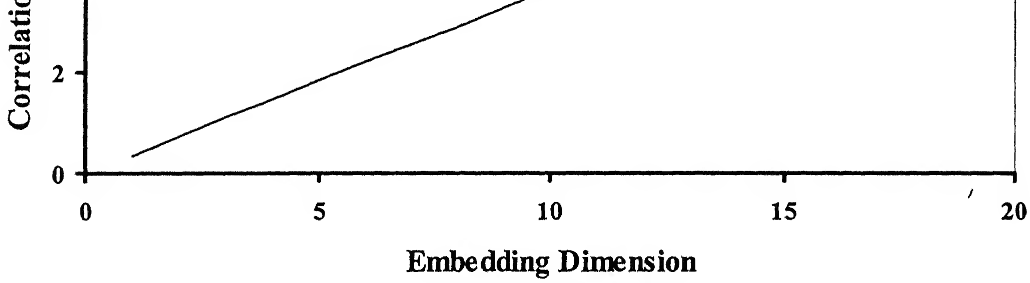


Figure 5.12.2: Correlation Exponent plot for 2-day Rainfall at Jackson

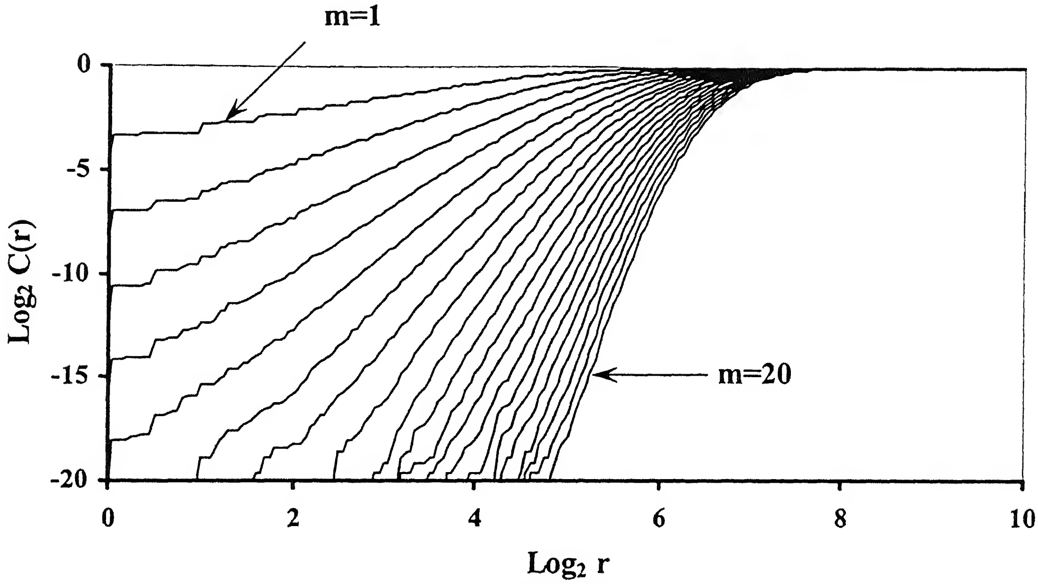


Figure 5.13.1: Correlation Integral plot of 5-day Rainfall at Jackson

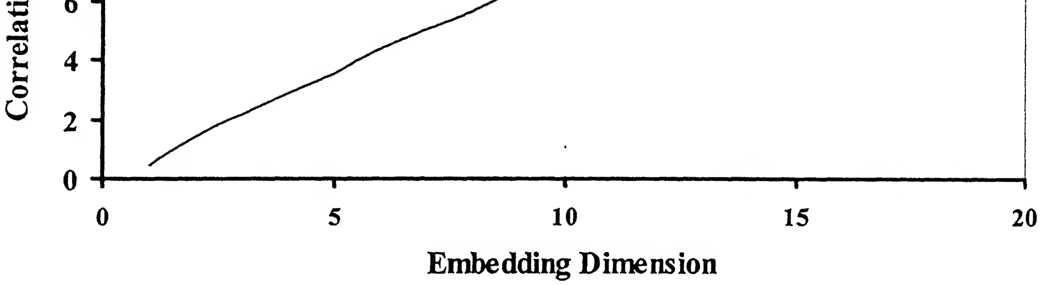


Figure 5.13.2: Correlation Exponent plot for 5-day Rainfall at Jackson

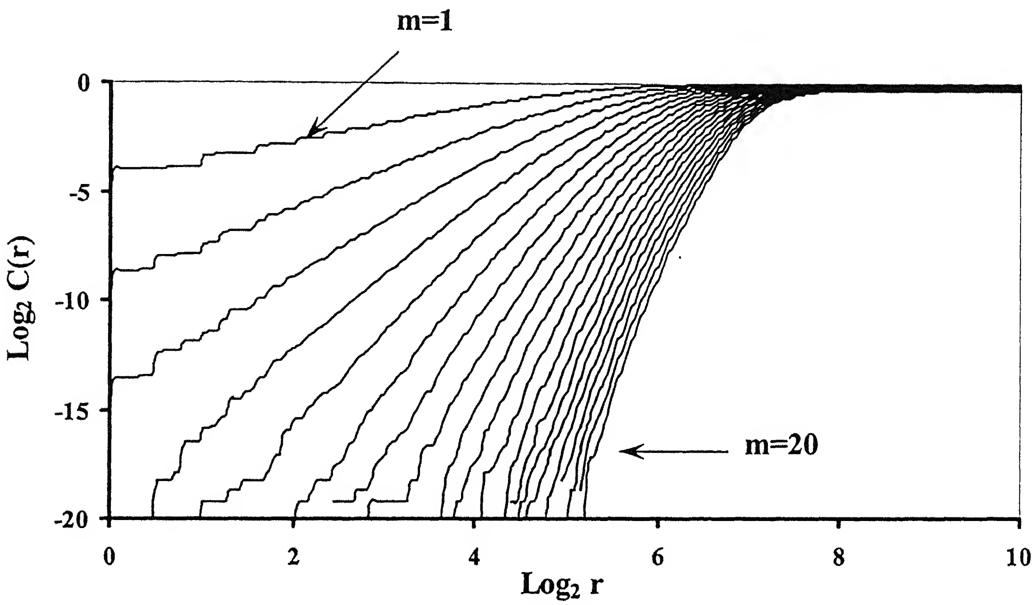


Figure 5.14.1: Correlation Integral plot of the 7-day Rainfall at Jackson

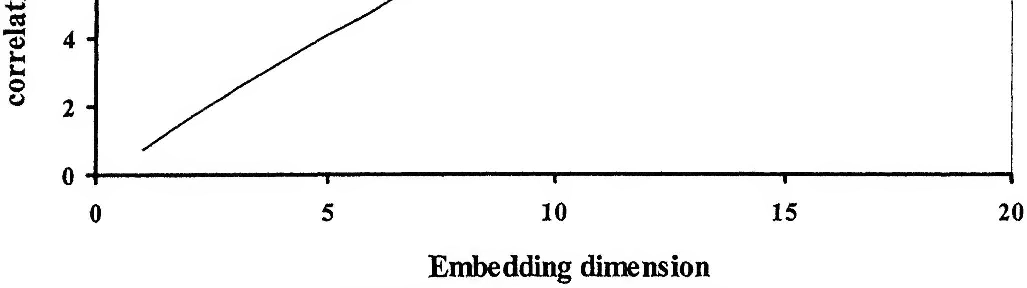


Figure 5.14.2: Correlation Exponent plot for 7-day Rainfall at Jackson

5.3.2 Scaling Effects in Flow Data

1-day, 5-day and 7-day data were analyzed for the purpose of investigating scaling effects in flow data at Ford Lock 10. The 5-day and 7-day flows were obtained by adding successive 5 and 7 daily flow values, respectively. Correlation Integral algorithm employed the original data, but for the calculation of the Lyapunov Exponent normalized data was used to simplify calculations. Results from correlation integral analysis for investigation of scaling effects for 1-day, 5-day, and 7-day flow data are presented in Figures 5.15.2 and 5.16.2 respectively. Lyapunov Exponent estimates are presented in the Table 5.4.

Figure 5.15.1 shows the relationship between Correlation function, C(r), and radius, r, for various embedding dimensions for the 5-day runoff at Ford Lock 10. Figure 5.15.2 shows the relationship between Correlation Exponent and embedding dimension values for the same data set. It can be observed from Figure 5.15.2 that the Correlation Exponent increase with increase in embedding dimension up to a certain point and saturates beyond that dimension. The saturation value of the Correlation Exponent and embedding dimension were found to be 4.08 and 11, respectively. Similar to daily flow and 5-day flow, 7-day flow has also shown the saturation of Correlation Exponent from embedding dimension 12 onwards. Saturation of the Correlation Exponent can be considered as sign for the existence of the chaos, and it provides the number of variables and minimum embedding dimension to embed the attractor. All these values are presented in the Table 5.3 for all the data sets.

From Table 5.8, Coefficient of variation suggests that higher resolution runoff series has the higher variability and vice-versa. It is contrary to what we have obtained by Correlation Integral results. The high resolution data i.e. daily runoff series has the lower Correlation Exponent and higher coefficient of variation and vice-versa. Similar results were obtained for the scaling effects in both rainfall series i.e. at London and Jackson. In this regard, it is not clear whether underestimation of the Correlation Exponent for the higher resolution data or overestimation of the Correlation Exponent for the lower resolution data is the cause showing for such a inverse relationship between coefficient of variation and Correlation Exponent.

The latter part i.e. over estimation (due to noise) might not be the reason because; here analyzed data is runoff, which is less prone to attack by noise unlike rainfall. In case of former reason i.e. under estimation (due to repeated occurrence of single value in the data e.g. Tsonis *et al.*, 1994) of the higher resolution data is also not possible. Like in rainfall data, it is not possible to have a single repeated variable in the different resolution of the runoff series. At this point it is hard to consider any of the above results unless it is firmly supported by other techniques.

However, Lyapunov Exponent method, with their positive largest Lyapunov Exponent values supports the existence of chaos. As observed in the case of rainfall data sets, the data set also showed inverse relationship between Lyapunov Exponent and Correlation Exponent values. And it is also observed that with decrease in the resolution of the data, Lyapunov Exponent also decreased. As we are not certain about the magnitude

Table 5. 9 Statistics of daily, 5-day, and 7-day flow data from the Kentucky River

Statistic	Daily (ft^3/s)	5-day (ft^3/s)	7-day (ft^3/s)
Number of data	10958	2191	1565
Mean	5462.32	27255.92	38158.28
Standard deviation	8592.35	38123.75	50659.64
Skew	3.662	3.058	2.824
Maximum value	99100	354000	481930
Minimum value	116	606	879
Coefficient of variation	1.573	1.398	1.328

Of the calculated Lyapunov Exponent, (because the calculated Lyapunov Exponents are selected one over a consistency range and moreover this method adopted just to check the existence of positive values only) it is difficult comment on the inverse relationship.

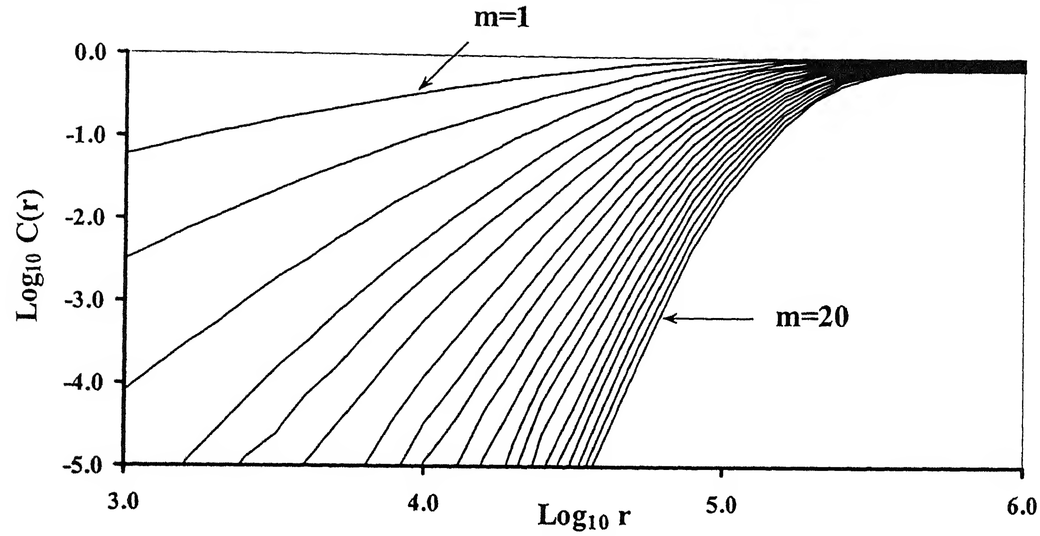


Figure 5.15.1: Correlation Integral plot of 5-day Flow at Lock10

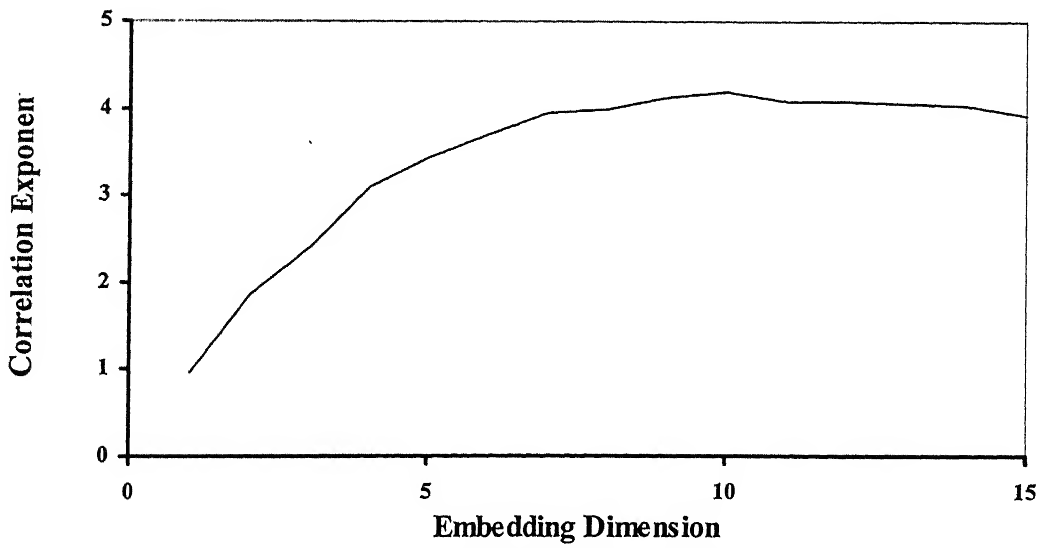


Figure 5.15.2: Correlation Exponent plot for 5-day Flow at Lock 10

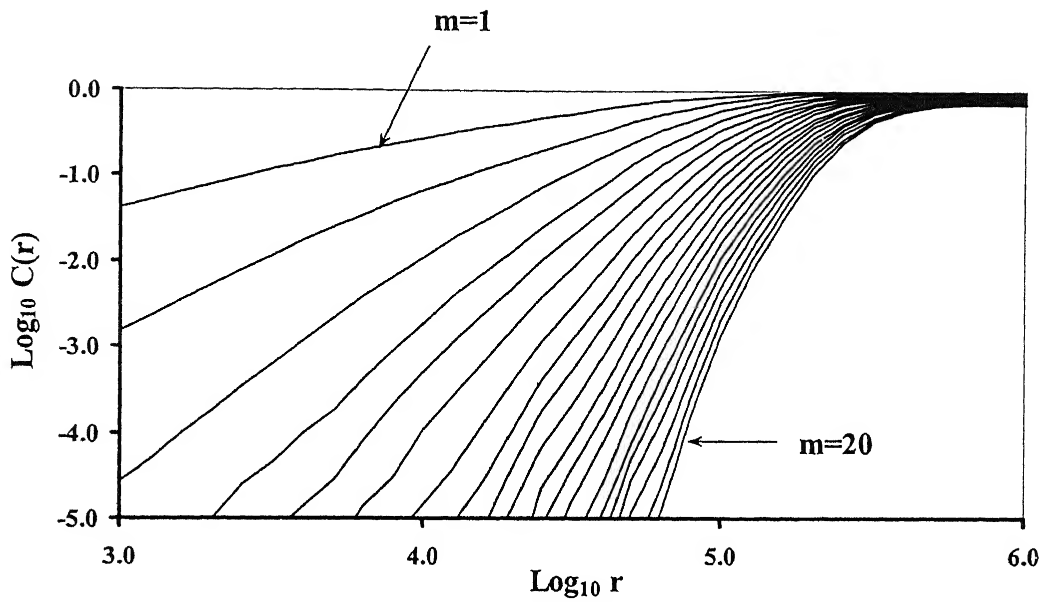


Figure 5.16.1: Correlation Integral plot of 7-day flow at Lock 10

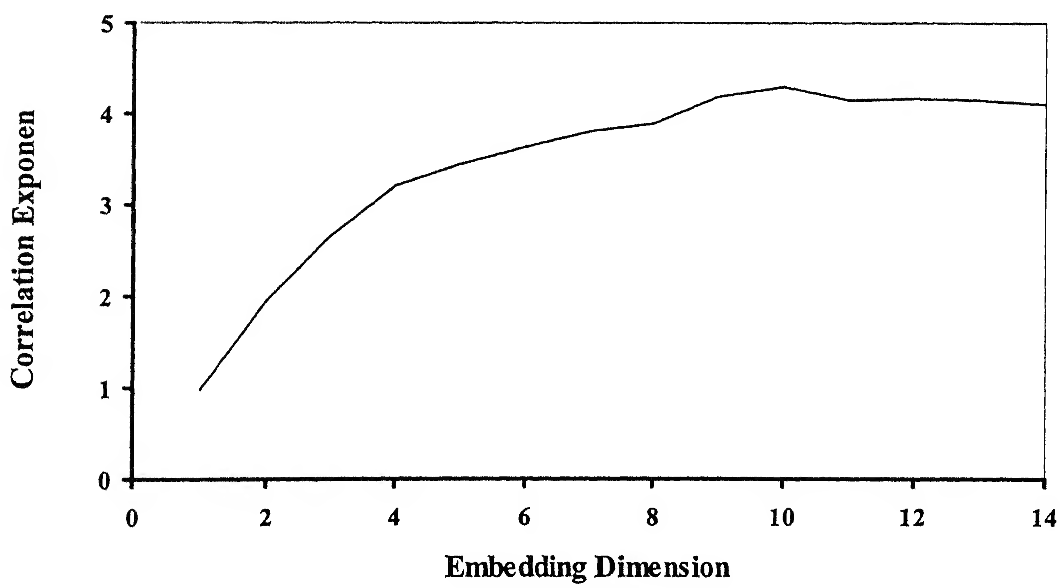


Figure 5.16.2: Correlation Exponent plot for 7-day flow at Lock 10

5.4 Results for Spatial Effects

5.4.1 Spatial Effects for Rainfall

To understand the spatial effects of the rainfall, we can average the daily rainfall of various raingauages scattered in a catchment. In the present case, it was achieved by averaging rainfall data from three stations, namely, Jackson, Hyden and Manchester. Then we employed both Correlation Integral and Lyapunov Exponent algorithms to investigate for the existence of the chaos. The results of this analysis are presented in Table 5.1, Table 5.2, Figure 5.17.1, and Figure 5.17.2.

Figure 5.17.2 shows the relationship between Correlation Exponent and embedding dimension for the averaged rainfall data at KRB. According to Porporato and Ridolfi (1996), the values of the Correlation Exponent and Embedding dimension were found to be 5.03 and 20. The following observations have been made 1) Similar to the daily rainfall data, spatially averaged daily rainfall also did not show saturation of the Correlation Exponent and 2) Value of the Correlation Exponent of average daily rainfall was higher than that for positive daily rainfall of the individual stations. Positive Lyapunov Exponents are obtained for the same data set.

Like other daily rainfall series, the above data give clues suggesting the existence of chaos. The value of the Correlation Exponent indicates that the average daily rainfall at KRB exhibits the highest variability. The value of Correlation Exponent for

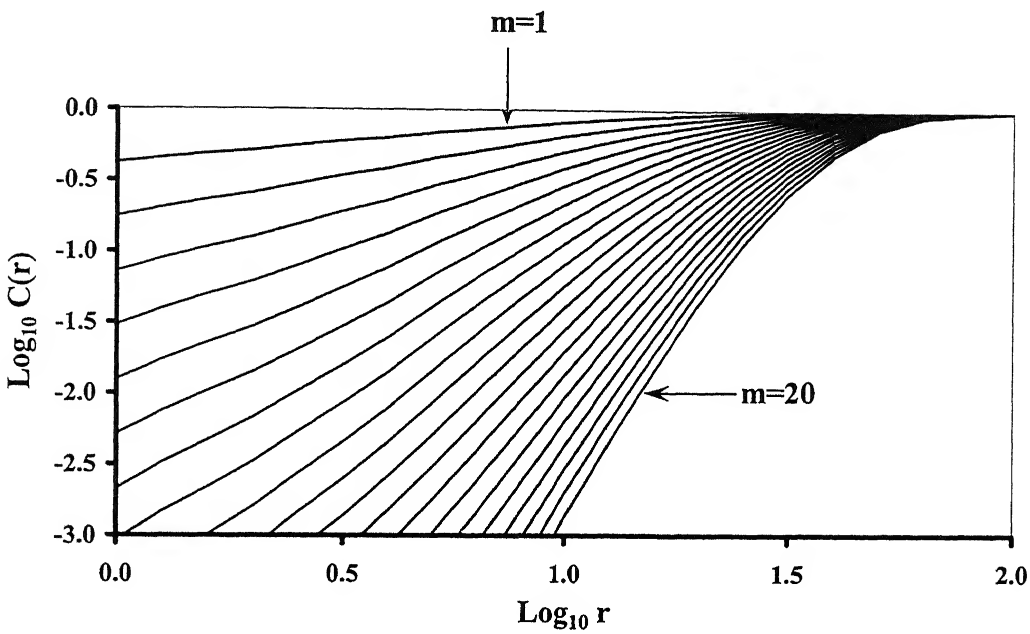


Figure 5.17.1: Correlation Integral plot of Average Daily Rainfall at upstream

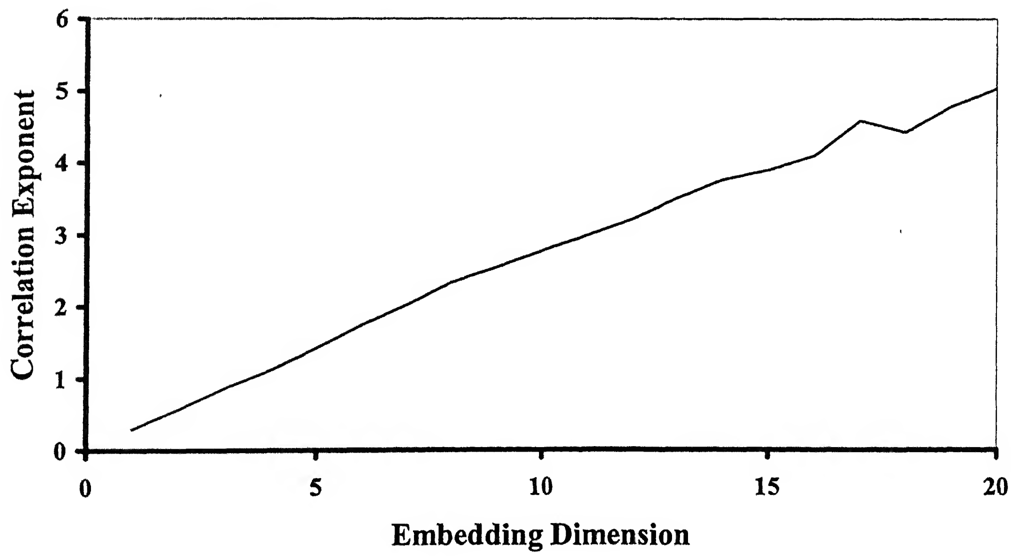


Figure 5.17.2: Correlation Integral plot for Average Daily Rainfall at upstream

daily rainfall at Jackson, Hyden, and Manchester are 3.98, 4.25, and 4.13, respectively; and that for the average rainfall from these locations is 5.03. That is to say that minimum number of physical variables needed to model daily rainfall at Jackson, Hyden and Manchester are four, five and five respectively; and that for the average rainfall from these three locations is five (assuming $5.03 \approx 5.00$). The minimum number of physical variables needed to model the spatially averaged hydrologic variable is equal to the maximum of the minimum number of physical variables needed to model the individual hydrologic variables employed for spatial averaging. However, the magnitude of the Correlation Exponent for the average rainfall series is higher than that for the individual rainfall series, which is not along the expected lines.

It can be said that Lyapunov Exponent method is conclusive as far as the existence of the chaos in Average daily rainfall data from the Kentucky River Basin is concerned, and existence of low-dimensional dynamics in daily rainfall data at Jackson, Hyden, and Manchester cannot be ruled out according to Porporato and Ridolfi (1996).

Comparing the results of the Correlation Exponent of runoff and rainfall at both upstream and downstream stations reveals that Correlation Exponent of the runoff at both stations is greater than that for the rainfall at the same stations. It appears that minimum numbers of variables needed (or degrees of freedom of the process) are more for runoff as compared to those for rainfall in the same area. It suggests that runoff process is more complex phenomenon than rainfall process. This is against the general opinion.

However, if we consider the upper limit of the number of variables (is the value of embedding dimension where correlation exponent saturates) needed to model a physical process then we find that rainfall process needs more number of variables as compared to those for runoff process (e.g. 20 v/s 13 and 10). The other possible explanation of such contrary results might be the criteria of saturation of ν v/s *m* plot for supporting the existence of the chaotic dynamics in physical process. As we have seen that the ν v/s *m* plot saturates clearly in case of runoff but does not saturate for rainfall.

5.4.2 Spatial Effects for Flow Data

Comparing the values of Correlation Exponent of average daily runoff at both upstream and down stream locations, reveals that value of the Correlation Exponent is less at downstream, Ford Lock 10, than upstream, Heidelburg. (*i.e,* 3.97 and 5.47 respectively). It suggests that the number of variables needed to model the average daily flow at Ford Lock 10 four is less than that needed to model the average daily flow at Heidelburg are six; also, flow at Ford Lock 10 have lower variability as compared to flow at Heidelburg. Also maximum number of physical variables needed to explain the daily average flow process at Lock 10 and Heidelburg are 10 and 13 respectively. These findings are in agreement with general belief that the flow at a downstream location is more developed process as compared to flow at an upstream location.

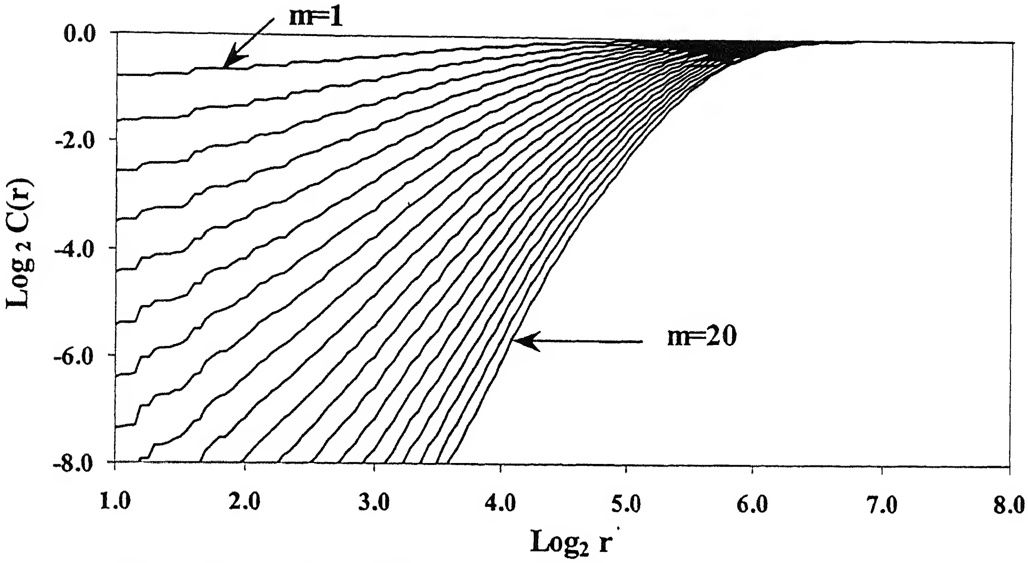


Figure 5.3.1: Correlation Integral plot of Daily Rainafll at Hyden

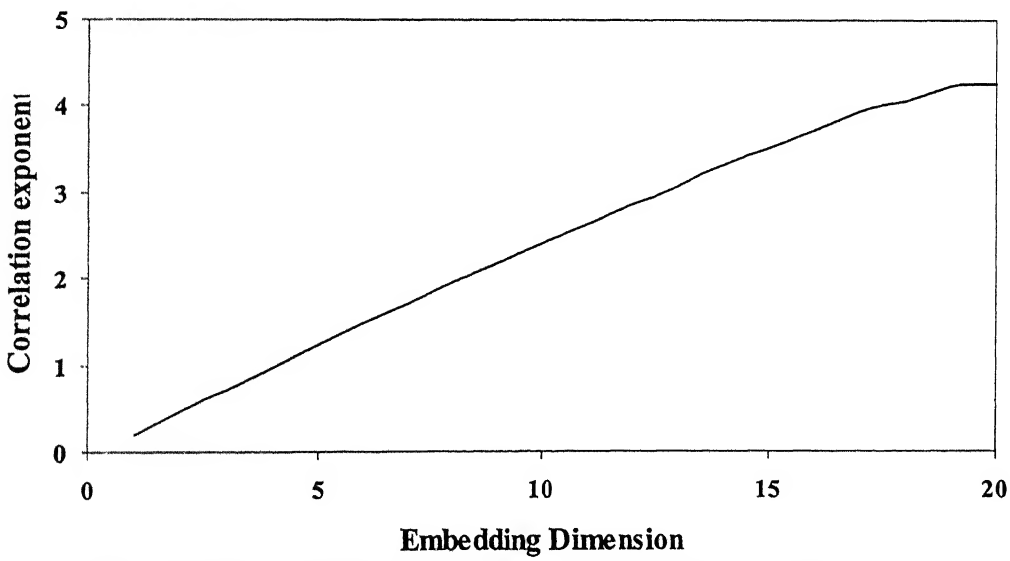


Figure 5.3.2: Correlation Exponent plot for Daily Rainfall at Hyden

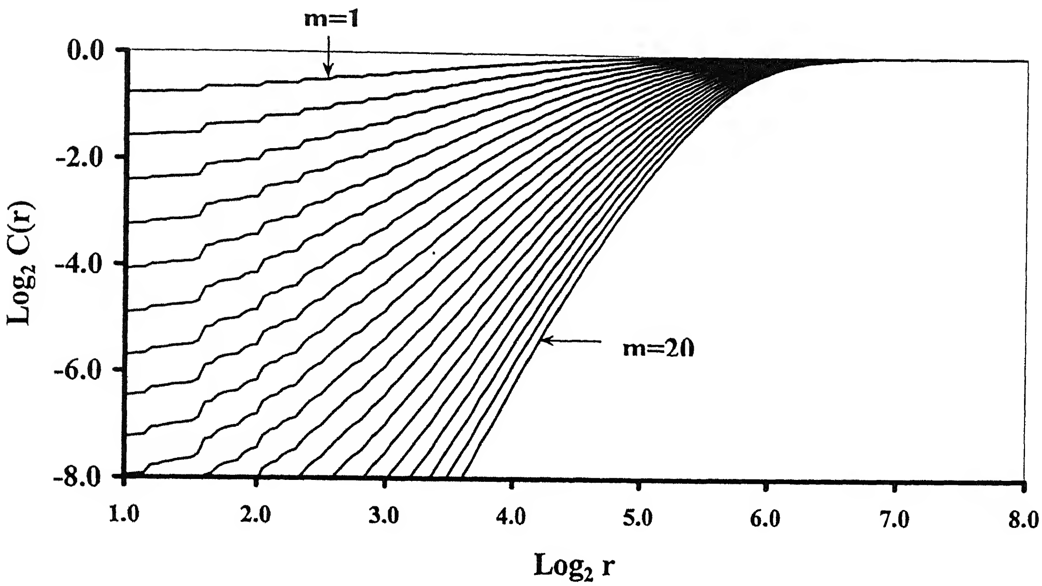


Figure 5.4.1: Correlation Integral plot of Daily Rainfall at Manchester

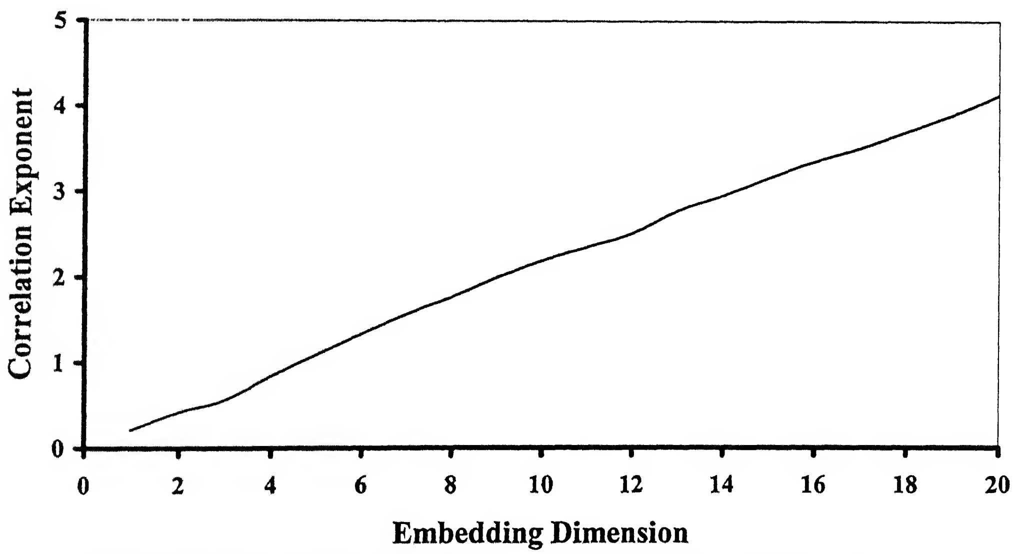


Figure 5.4.2: Correlation Exponent plot for Daily Rainfall at

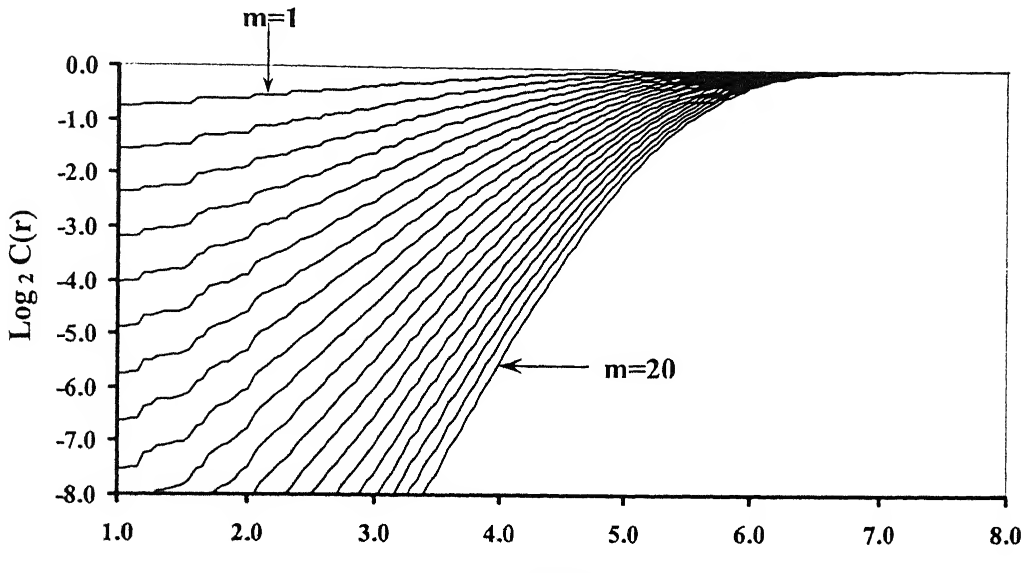


Figure 5.5.2: Correlation Integral plot of Daily Rainfall at Heiedelburg

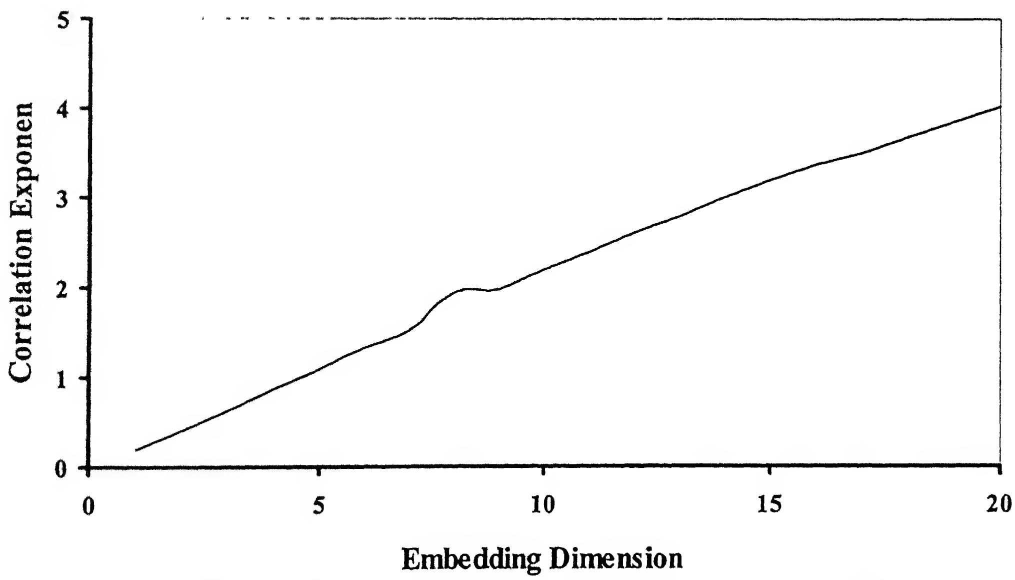


Figure 5.5.2 Correlation Exponent plot for Daily Rainfall at Heidelburg

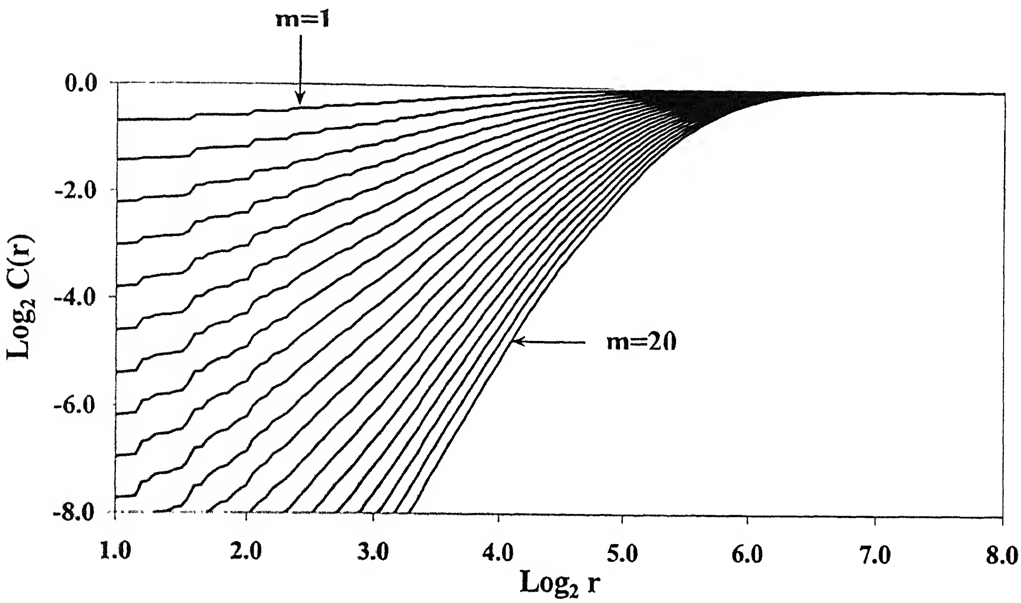


Figure 5.6.1: Correlation Integral plot of Daily Rainfall at Ford lock

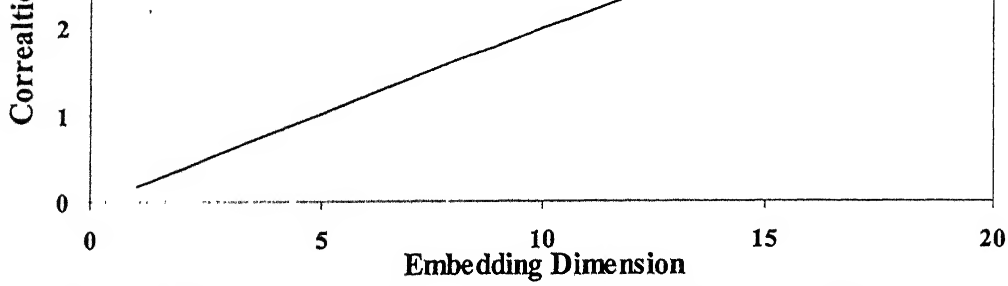


Figure 5.6.2: Correlation Exponent plot for Daily Rainfall at Ford lock 10

Chapter 6

CONCLUSIONS

This study presents the findings of an extensive investigation of deterministic dynamics in hydrologic variables from a chaotic perspective. The hydrologic variables investigated include total daily rainfall and average daily flow. The data derived from Kentucky River Basin (KRB) and North Fork Kentucky (NFKY) River Basin, a part of the KRB, were employed in this study. Specifically, total daily rainfall (mm) at London, Kentucky from NFKY river basin; total daily rainfalls (mm) from Jackson, Hyden, Manchester, Heidelberg, and Lexington from the KRB; and average daily flow (cfs) at Heidelberg and Lock 10 stations from KRB were employed. The techniques used to investigate for chaotic bahaviour in the hydrologic variables include Correlation Integral method and the Lyapunov Exponent method. The Correlation Integral method not only provides clues as to the existence of chaos in a data set but also gives information on the necessary and sufficient number of physical variables needed to model a physical process under investigation. The Lyapunov Exponent method provides information only on the existence of chaos in a data set. The Lyapunov Exponent was calculated over a consistency range, probably for the first time, as suggested by Rodriguez-Iturbe *et al.*, (1989).

The results obtained in the current study in terms of the Lyapunov Exponent strongly support the existence of chaos in both rainfall and flow data at all the locations investigated in the present study. Based on the Correlation Exponent method results, a strong evidence of existence of chaotic dynamics was found in the average daily flow data at both the locations investigated in the present study, and the existence of low dimensional deterministic dynamics can not be ruled out in the rainfall data at all the locations considered in the present study, according to Porporato and Ridolfi (1996).

Daily Rainfall at London, Jackson, and Ford Lock 10 needs a minimum of four variables and Daily Rainfall at Hyden, Manchester, and Heidelburg need a minimum of five variables to model the dynamics of the rainfall at these locations, respectively. Average Daily Rainfall from Jackson, Hyden, and Manchester need a minimum of five variables to model the process. Based on the attractor dimensions that resulted, the minimum number of variables essential to model the average daily flow process at Heidelburg and Lock 10 of KRB are 6, and 4, respectively, whereas the number of variables sufficient are 13 and 10 respectively. Minimum number of variables essential to model the 5-day and 7-day flow processes of Lock 10 were found to be five and the number of variables found sufficient are 11 and 12, respectively.

The current study also made an attempt to investigate for the temporal scaling effects in both rainfall and flow data, and spatial effects in both rainfall and flow data from the Kentucky River basin. The temporal scaling effects in rainfall data were

investigated for one, two, five, and seven days; whereas, the temporal scaling effects in flow data were investigated for one, five, and seven day resolutions. The value of an n-day data series was computed by simply adding the successive n values of the particular time series. The investigations of 2-, 5-, and 7-day rainfall data, and 5-day and 7-day flow data, have revealed results similar to those for the 1-day rainfall and 1-day flow data, respectively. That is, the scaling effects for a river flow process have also been found to be chaotic; whereas, the existence of low dimensional chaotic dynamics in scaling effects for the rainfall process from the KRB can not be ruled out. Further, both coefficient of variation and Correlation Exponent are measures of variability in a data set. However, the computation of these two for different resolution rainfall data reveals that if one of them increases then the other decreases with a decrease in the resolution of the rainfall data. In the present case, the value of coefficient of variation decreases from 2.39 to 0.92 as the resolution of the rainfall decreases from a day to seven days. On the contrary, the Correlation Exponent increases from 3.92 to 11.09 with the decrease in the resolution from 1-day to 7-days (see Table 5.5). The results of coefficient of variation are on the expected lines but those for the correlation exponent are not. The reason for such contrary results may be the different degrees of noise levels and the presence of large number of zeros in the rainfall data series that lead to an underestimation of the correlation exponent for higher resolution data and overestimation of the correlation exponent for the low resolution data. These results obtained in the present study verify the earlier studies reported by Tsonis *et al.*, 1994 and Sivakumar, 2001 for the rainfall data. However, the Lyapunov exponent behaves in a manner similar to the coefficient of variation with respect to the

temporal scaling effects in rainfall data. This indicates that the Lyapunov exponent method may be more reliable than the correlation integral method as the latter is more sensitive to the presence of zeros, level of noise in the data, and is not as robust as the Lyapunov exponent method.

The investigation of spatial effects in both rainfall and flow data investigated in the current study have shown that the nature of the deterministic dynamics is not dependent on space; however, the number of physical variables that are necessary and sufficient to model a physical process at one location in a basin may differ from those at another location of the same basin. For example, in case of spatial effects for the rainfall data from KRB, the results exhibit a similar trend for individual rainfall data series and the spatially averaged rainfall series from three locations (Jackson, Hyden, and Manchester) for both correlation integral method and Lyapunov exponent method. The values of the Correlation Exponents for various daily rainfall data series from the KRB range from 3.83 (for London) to 4.24 (for Hyden). The Correlation Exponent value for the spatially averaged daily rainfall data series (5.03) was found to be more than those for all the three individual daily rainfall data suggesting that spatial averaging of daily rainfall data series actually complicates a physical process to some extent as far as modeling of the physical process is concerned. However, the minimum number of physical variables needed to model the spatially averaged hydrologic variable is equal to the maximum of the minimum number of physical variables needed to model the individual hydrologic variables employed for spatial averaging. The value of Correlation Exponent for the daily flow

series at a downstream location (Lock 10) was found to be less than that for an upstream location (Heidelberg) of the KRB suggesting that the physical process in the downstream reaches of a basin is probably less complex and fully developed as compared to that for the upstream reaches of the same basin.

Limitations and Scope for Future Work

The estimation of the largest Lyapunov exponent was computed using the method proposed by Wolf *et al.,* (1985), which is good for investigating a time series of physical variable for the existence of deterministic dynamics; however, it may not be desirable from interpreting the Lyapunov Exponent results from limit of predictability point of view. This leaves some scope of improvement of the present research effort.

The results obtained in the current study indicated an interesting observation while analysing the Correlation Integral and Lyapunov Exponent values for the daily flow data from the two locations of the KRB investigated in this study. The results suggest an inverse relationship between the magnitudes of Correlation Exponent and the Lyapunov Exponent. This inverse relationship was supported when the results obtained in the current study were analysed in conjunction with those taken from some other studies (e.g. Boringdon and Lisi,1999). There is no study that gives any guidelines to interpret the magnitude the Lyapunov exponent. Further, research work is needed in this area to interpret the Lyapunov exponent results and their relationship with the correlation exponent for the same data set.

Another interesting observation made while analysing the temporal scaling effects in daily rainfall data from the KRB, it was that the slope of the correlation exponent plot (*i.e.* correlation exponent versus embedding dimension) increases with a decrease in the resolution of the rainfall data. That is to say, the slope of the correlation exponent plot for a 7-day rainfall data is more steep than that for the 2-day rainfall, and the slope of the correlation exponent plot of the 2-day rainfall data is more than that for the 1-day rainfall data. The reason for this is not clear at this stage. However, one possibility might be the reduction in the total number of data points available for a low-resolution data series, that leads to a curtailed scaling region from which correlation exponent for a particular embedding dimension is computed. The curtailment in the scaling region may lead to the underestimation of the Correlation Exponent (e.g. Havstad and Ethlers, 1989). Another Possibility might be the presence of large number of single-valued observations in the data (Tsonis *et al.,* 1994). For example presence of large number of zeros in the daily rainfall data. However, these findings are based on the results obtained in the current study. Further investigations need to be carried out on data derived from other river basins to verify these findings.

The value of coefficient of variation was found to decrease with a decrease in the resolution of the daily runoff data but the Correlation Exponent exhibited an opposite trend *i.e.* it increases with a decrease in the resolution of the flow data. The contrary results in terms of the correlation exponent for the flow data can not be explained by high noise levels or the presence of large number of zeros (or any other single value) in

the flow data. More work needs to be carried out in this direction to investigate further as to the possible reasons for such contrary results of the correlation exponent for flow data at different scales. However, the Lyapunov exponent results are in agreement with the coefficient of variation for different resolution flow data. This raises doubts about the validity and applicability of the Correlation Integral method and the interpretation of the results especially in cases when the correlation exponent plot does not saturate.

Even though a lot of work has been carried out in the area of investigation of chaos in hydrology using the Correlation Integral method, there is no clear cut guideline for the selection of the radius 'r' needed to calculate correlation function C(r). The selection of radius values is an important step in the Correlation Integral method as it can affect the computation of Correlation Exponent, and the interpretation of the results ultimately. It is hoped that further research efforts in developing the clear cut guidelines for the selection of radius will be carried out for easy and more meaningful Correlation Integral analysis.

Finally, Porporato and Ridolfi (1996) have suggested that a milder slope in the correlation exponent plot may indicate the existence of the low dimensional deterministic dynamics in the data, in the absence of the saturated region in the correlation exponent versus embedding dimension plot. However, it is not clear what magnitude slopes can be considered milder, and what magnitude slopes can be considered steeper, as was the case in the present study, while analysing the temporal scaling effects in the total

daily rainfall data from KRB. It is hoped that certain guidelines can be developed that will help characterise a particular slope as a mild one or a steep one that will help interpret the results of the correlation integral method.

APPENDIX A

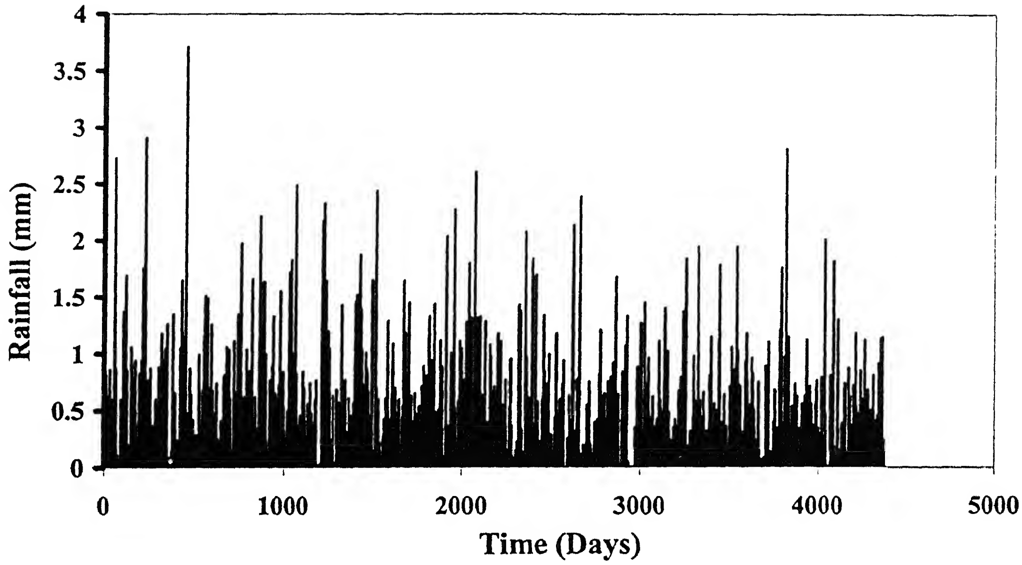


Figure A.1: Time series plot of Daily Rainall at London

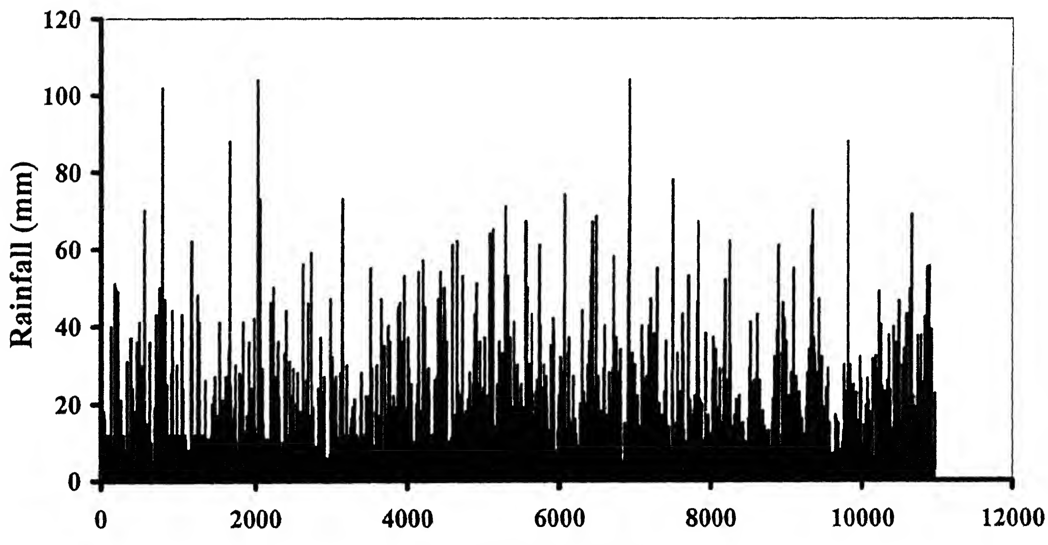


Figure A.2: Time series plot of Daily Rainfall at Jackson

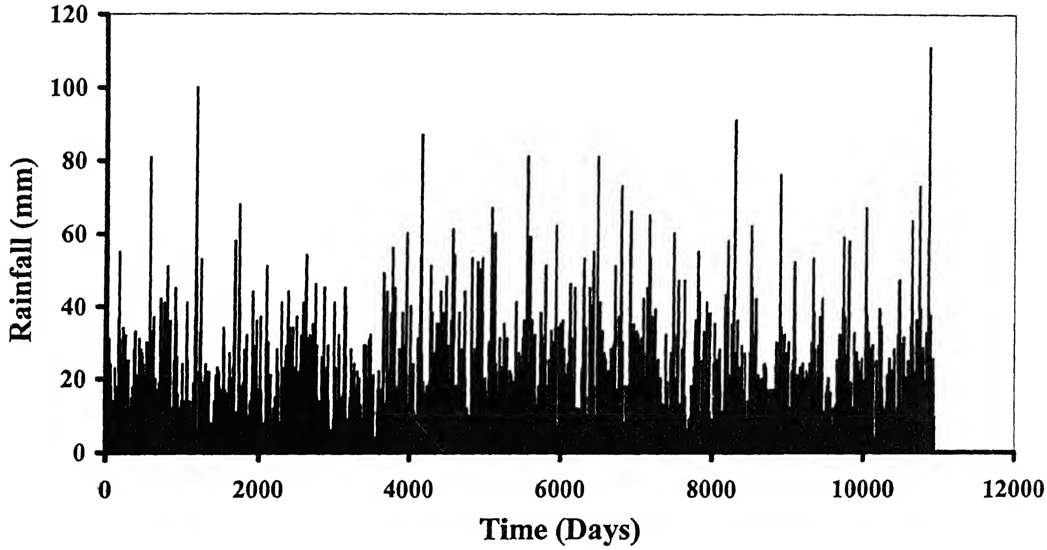


Figure A.3: Time series plot of Daily Rainfall at Heidelburg

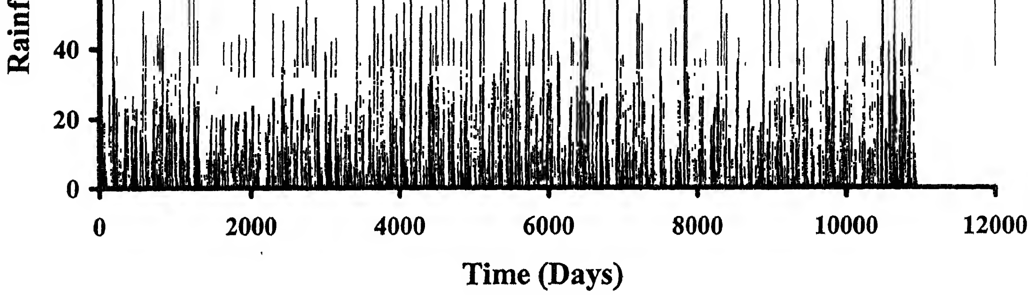


Figure A.4: Time series plot of Daily Rainfall at Manchester

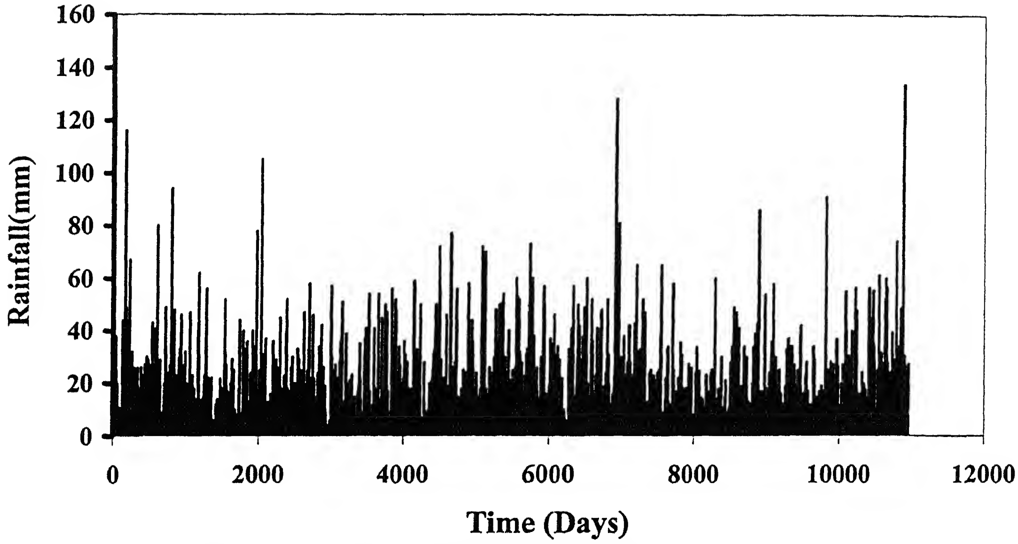


Figure A.5: Time series plot of Daily Rainfall at Heidelburg

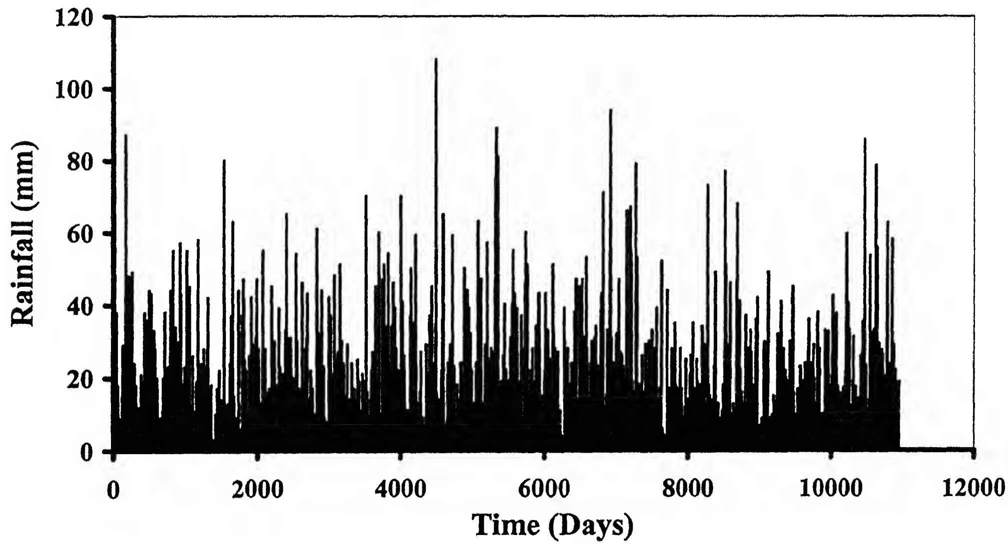


Figure A.6: Time Series plot of Daily Rainfall at Ford Lock 10

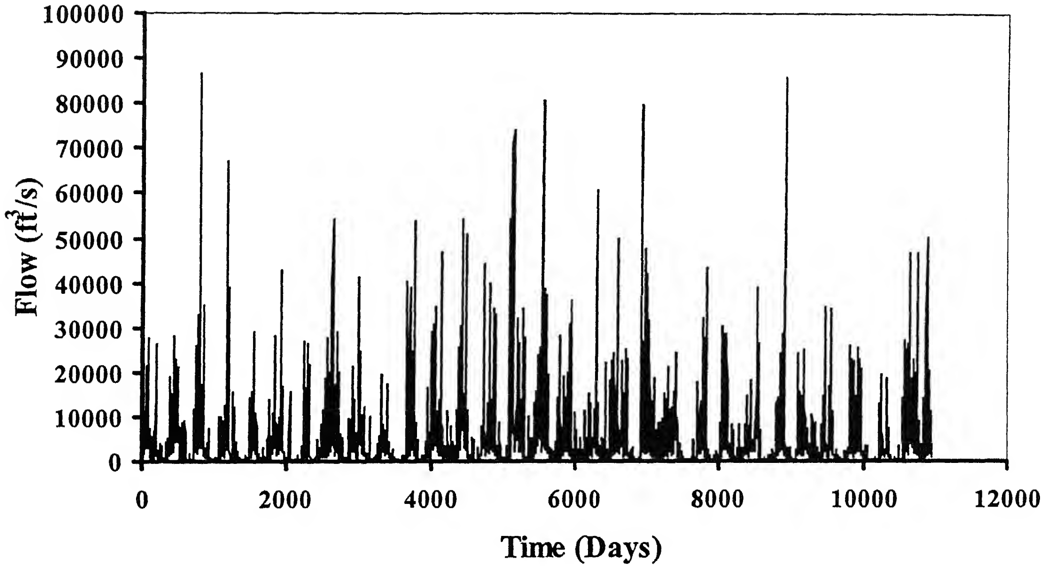


Figure A.7: Time series plot of Average Daily Flow at Heidelburg

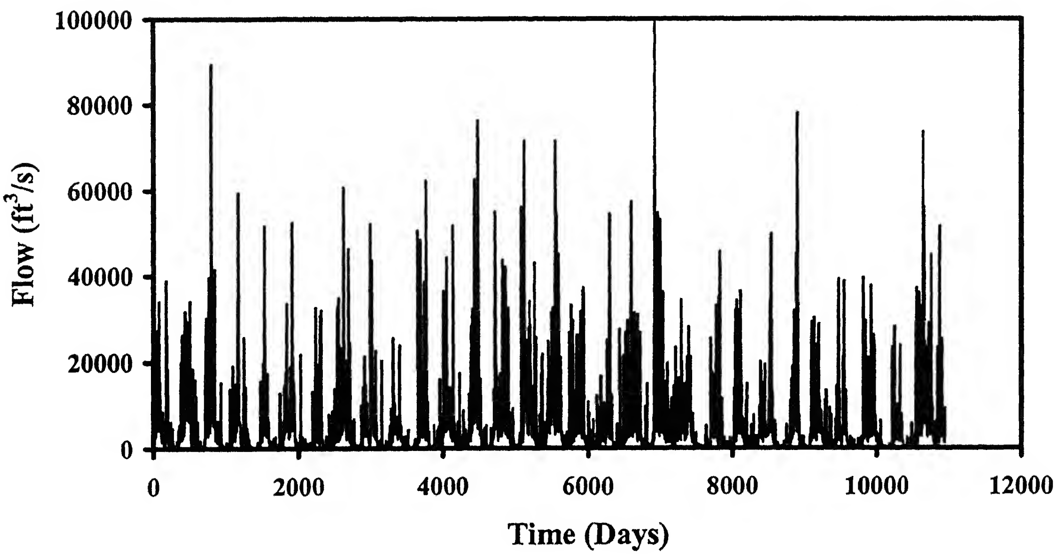


Figure A.8: Time series plot of Average Daily Flow at Lock 10

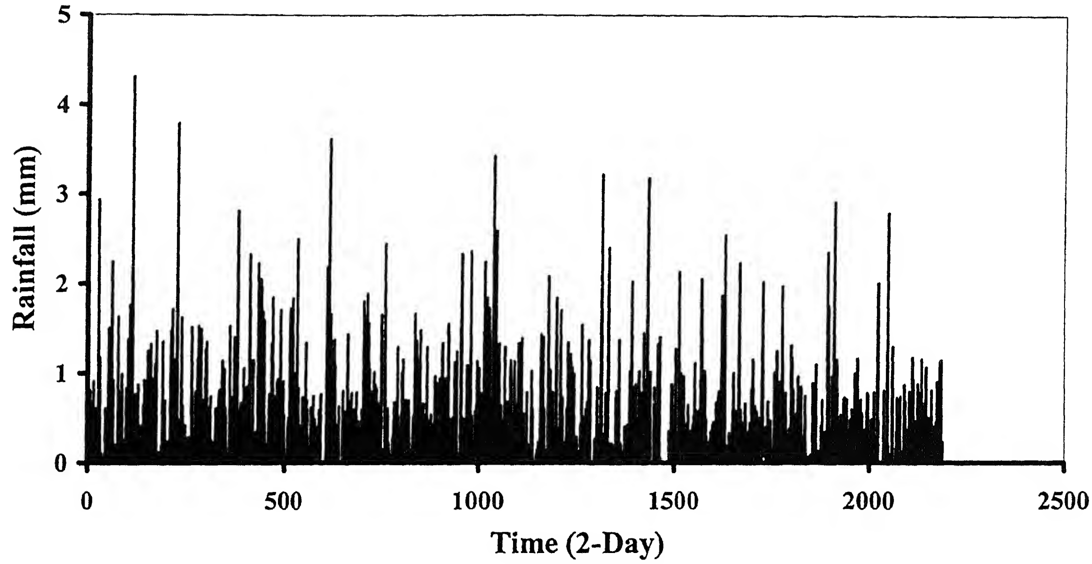


Figure A.9: Time series plot of 2-Day Rainfall at London

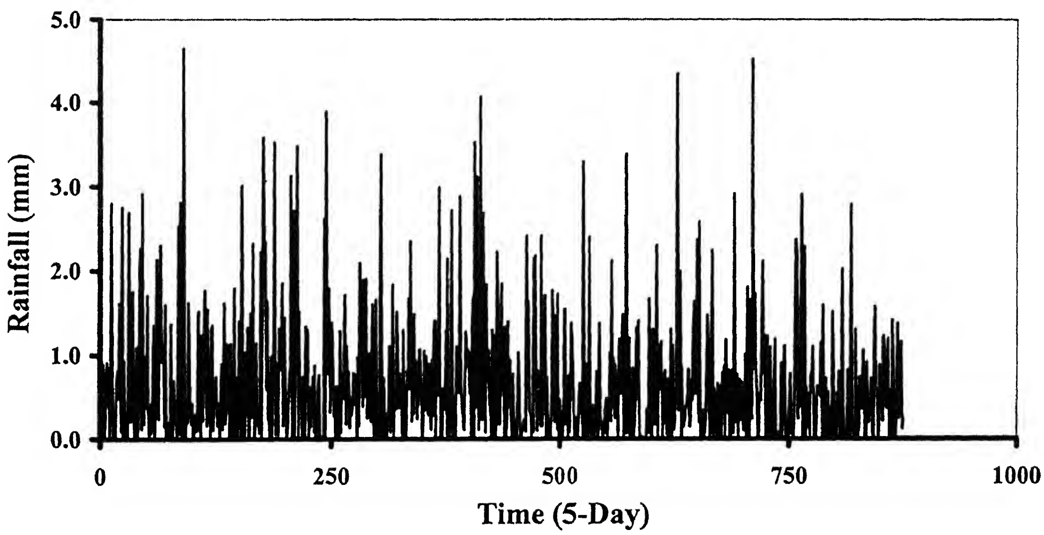


Figure A.10: Timeseries plot of 5-Day Rainfall at Lonodon

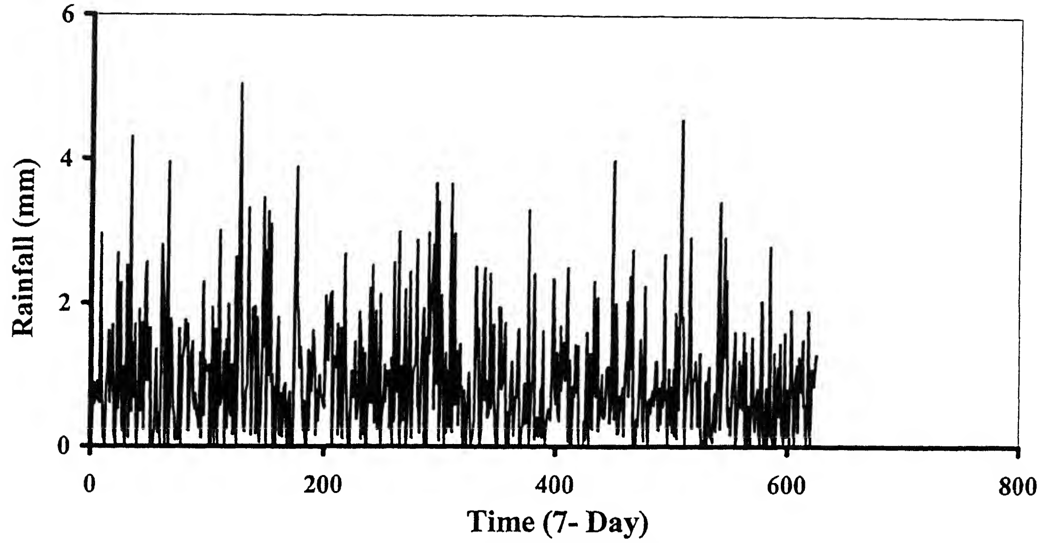


Figure A.11: Time series plot of 7-Day Rainfall at London

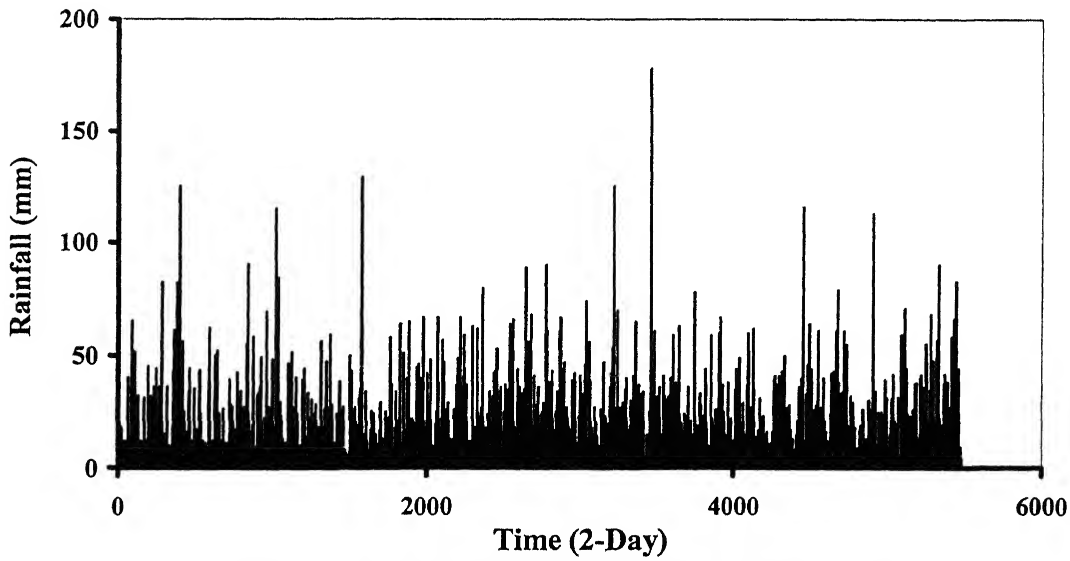


Figure A.12: Time series plot of 2-Day Rainfall at Jackson

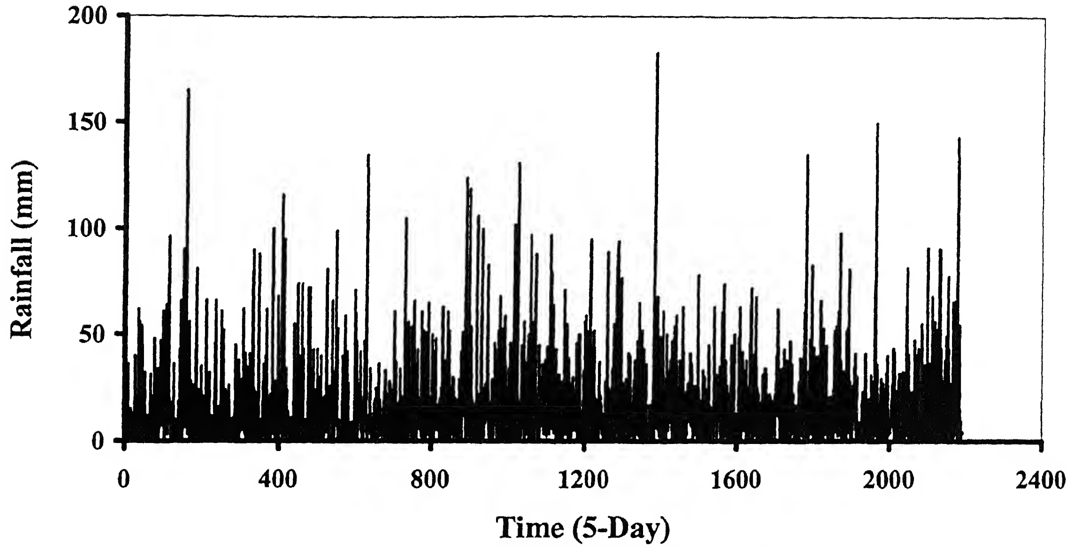


Figure A.13: Time series plot of 5-Day Rainfall at Jackson

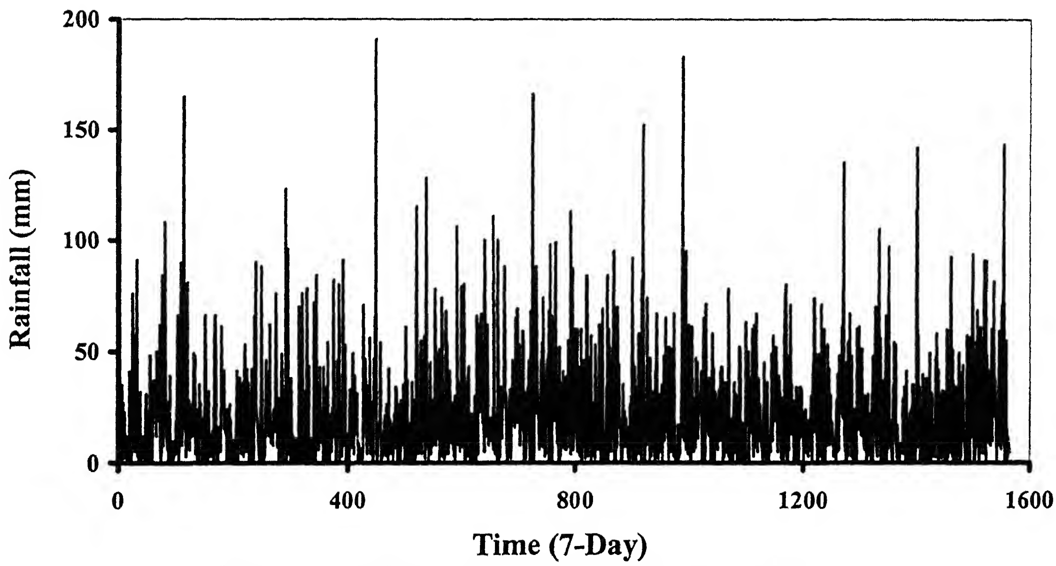


Figure A.14: Time series plot of 7-Day Rainfall at Jackson

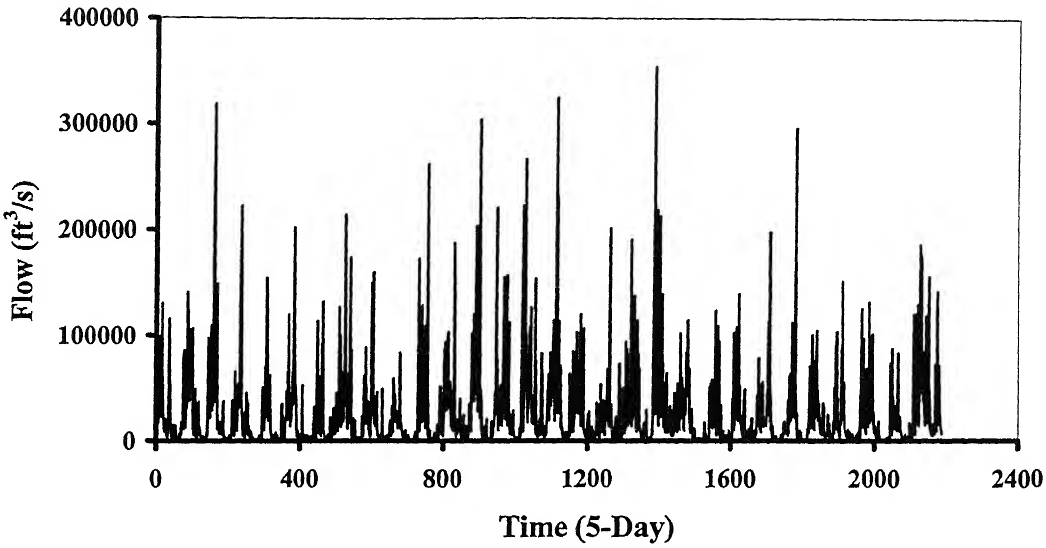


Figure A.15: Time series plot of 5-Day Flow at Lock 10

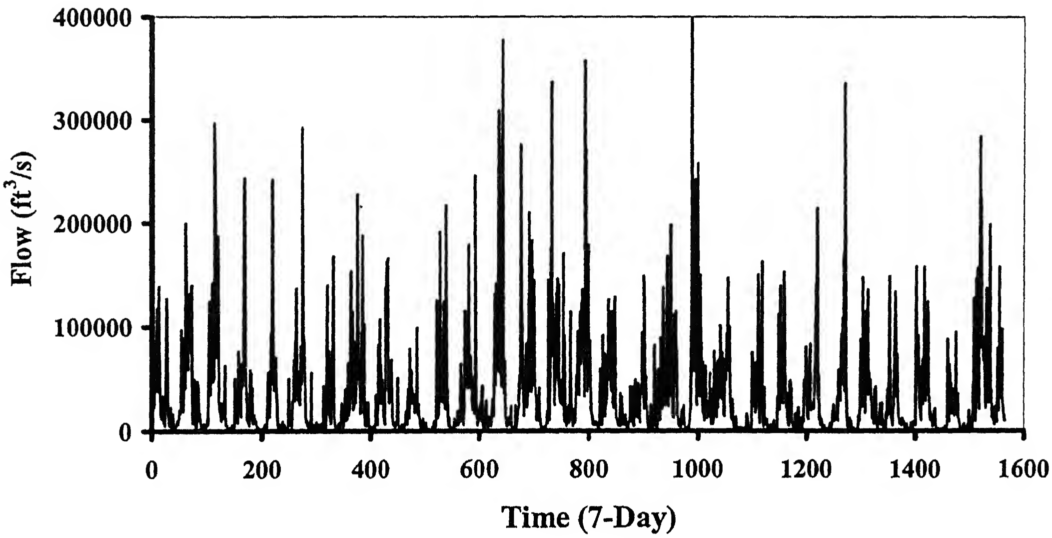


Figure A.16: Time series plot of 7-Day Flow at Lock 10

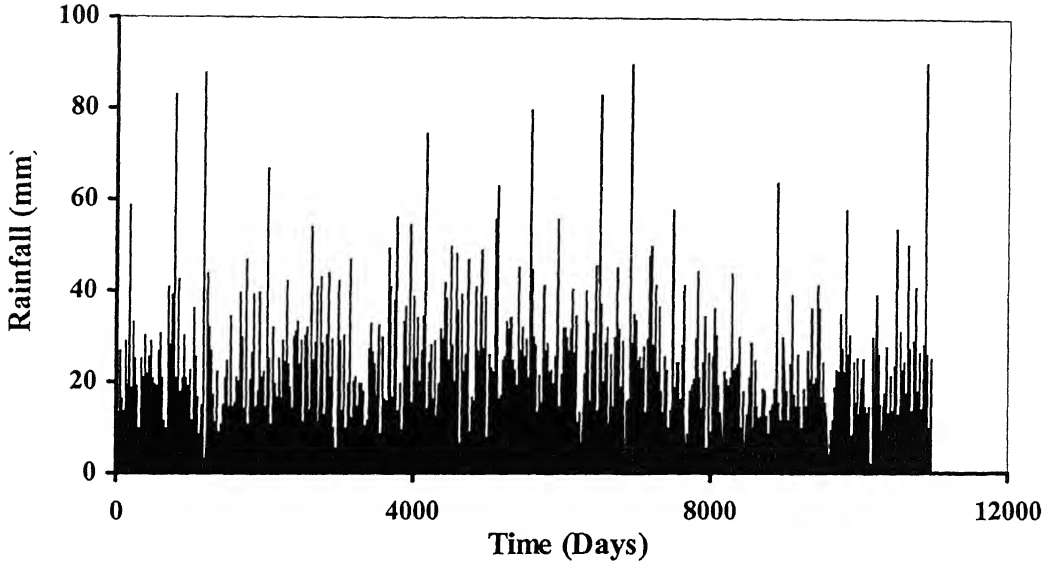


Figure A.17: Time series plot of Average Daily Rainfall at upstream

APPENDIX B

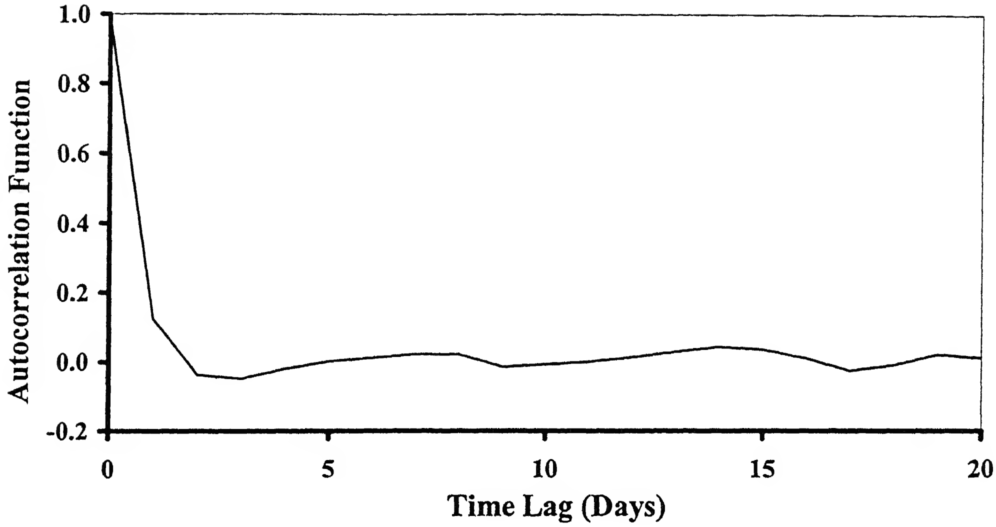


Figure B.1: Correlogram of Daily Rainfall at London

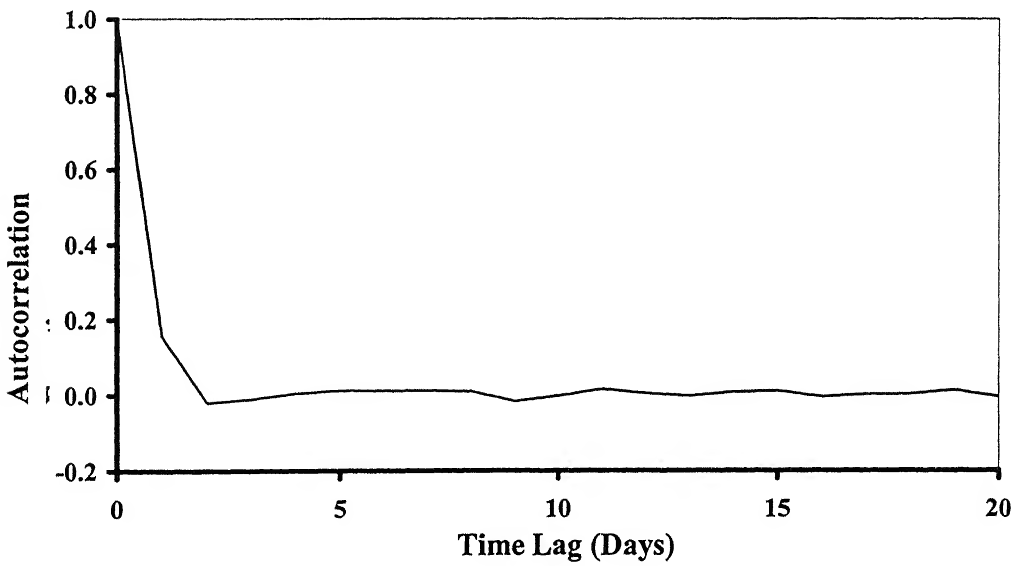


Figure B.2: Correlogram of Daily Rainfall at Jackson

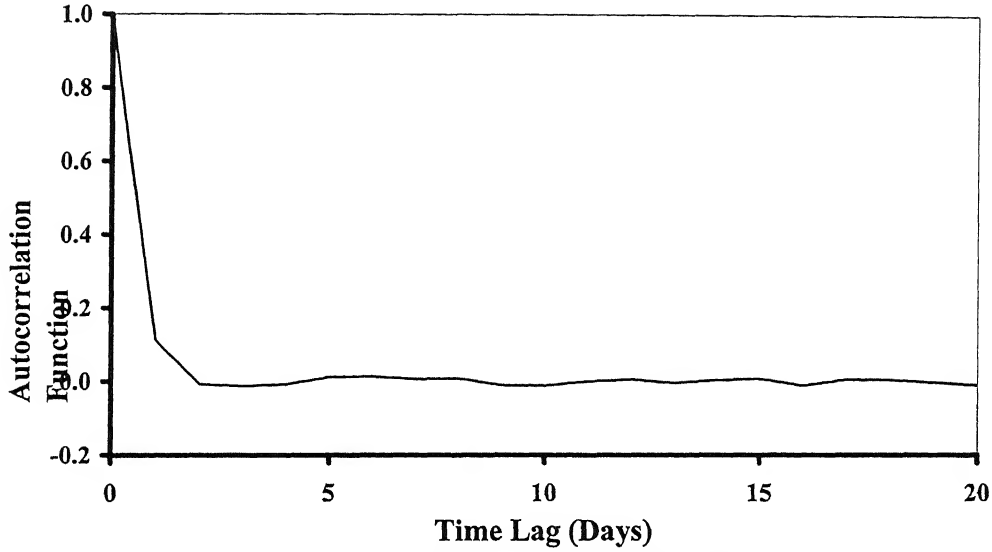


Figure B.3: Correlogram of Daily Rainfall at Hyden

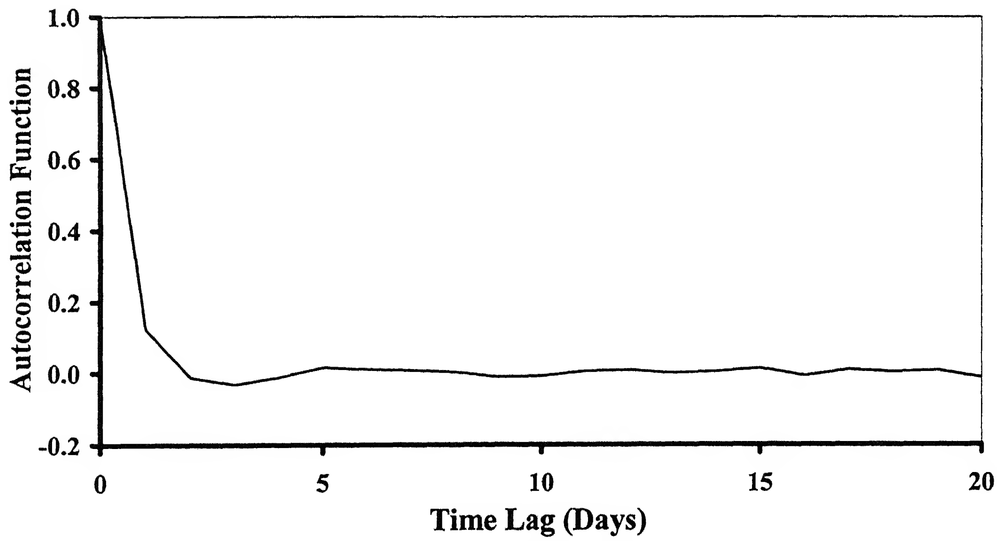


Figure B.4: Correlogram of the Daily Rainfall at Manchester

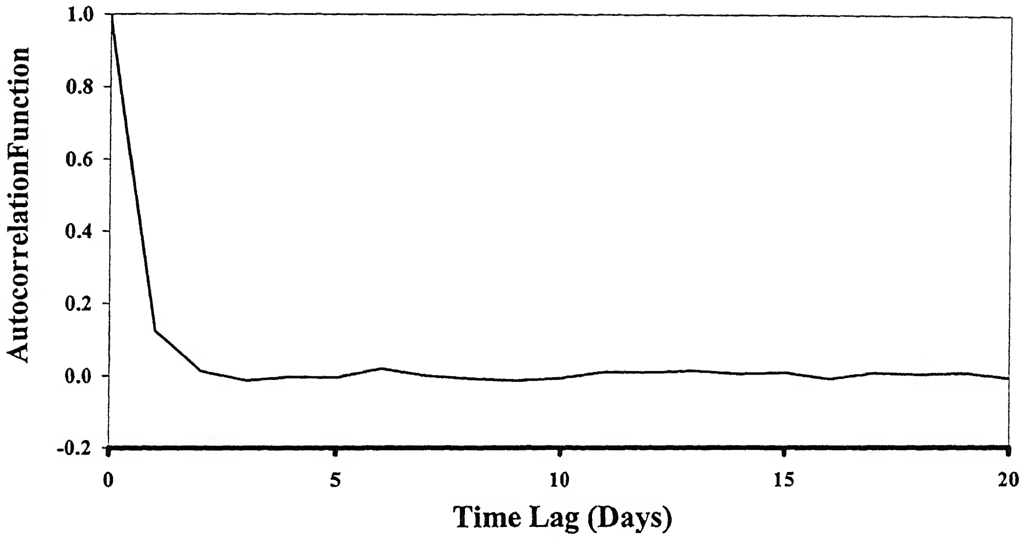


Figure B.5: Correlogram of Daily Rainfall at Hyden

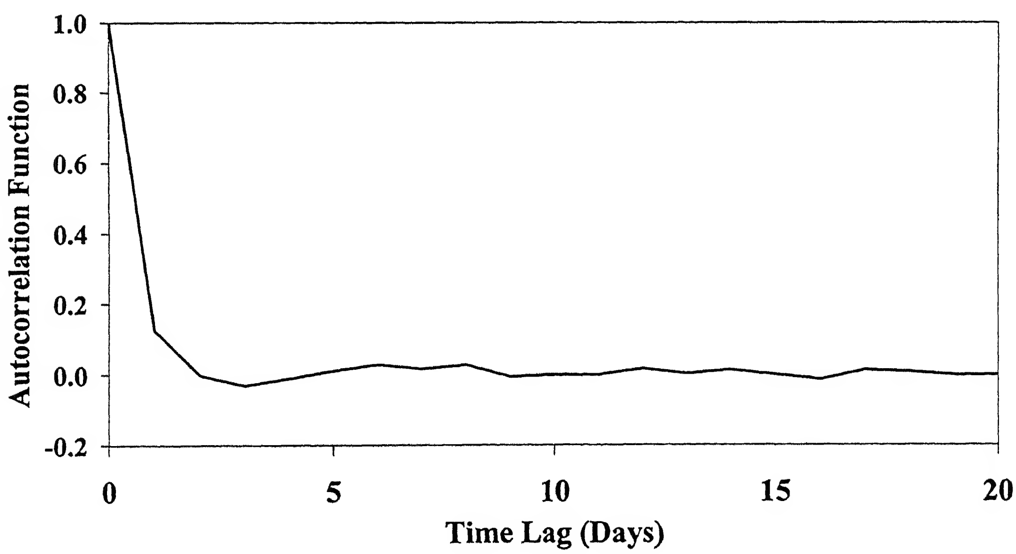


Figure B.6: Correlogram of Daily Rainfall at Ford Lock 10

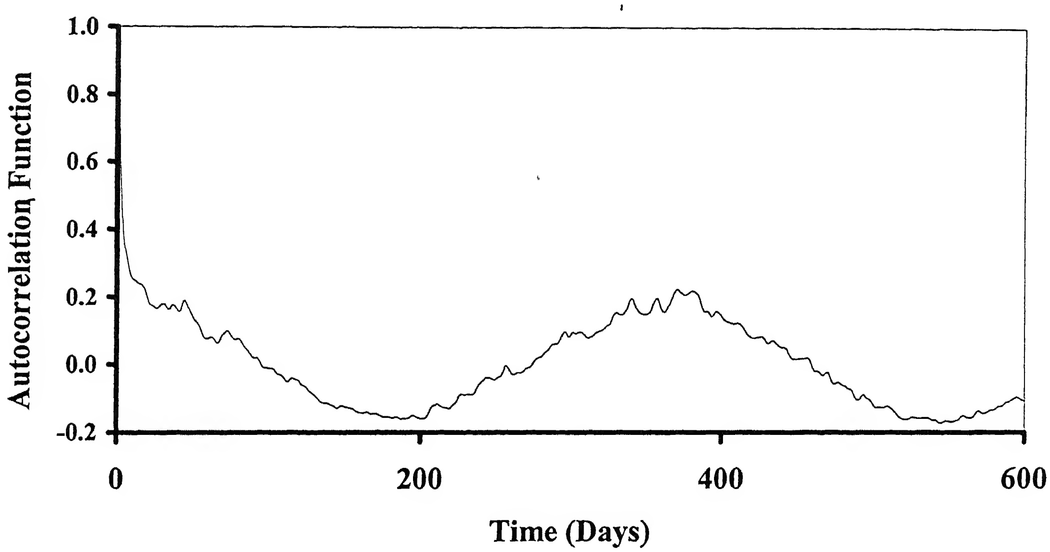


Figure B.7: Correlogram of Average Daily Flow at Heidelburg

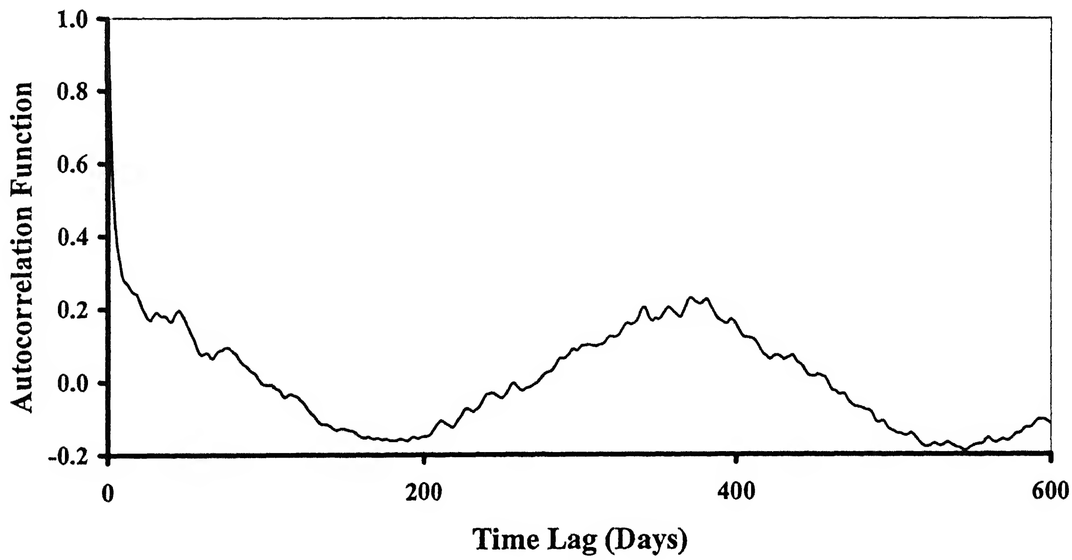


Figure B.8: Correlogram of Average Daily Flow at Lock 10

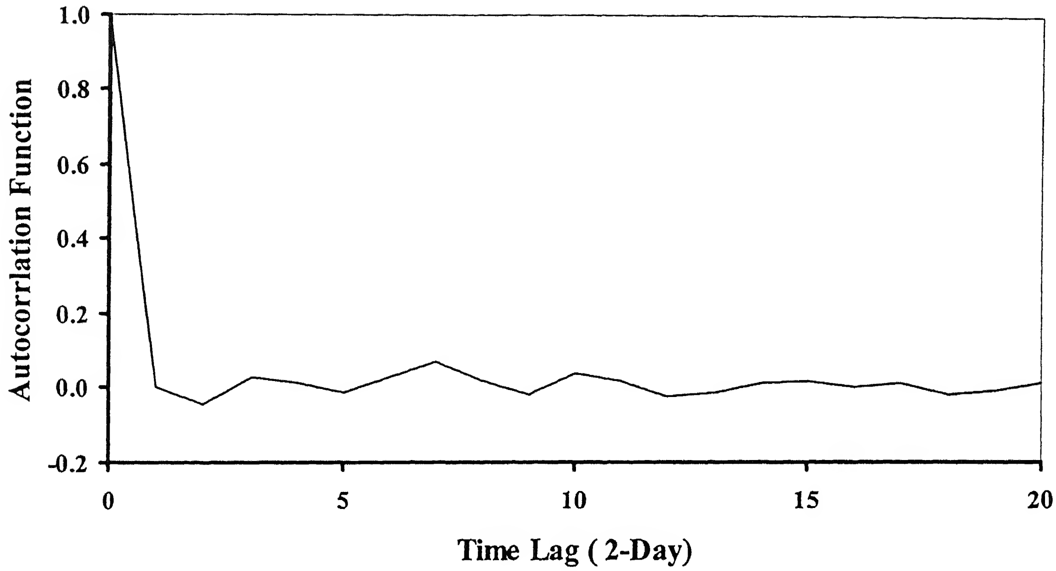


Figure B.9: Correlogram of 2-Day Rainfall at London

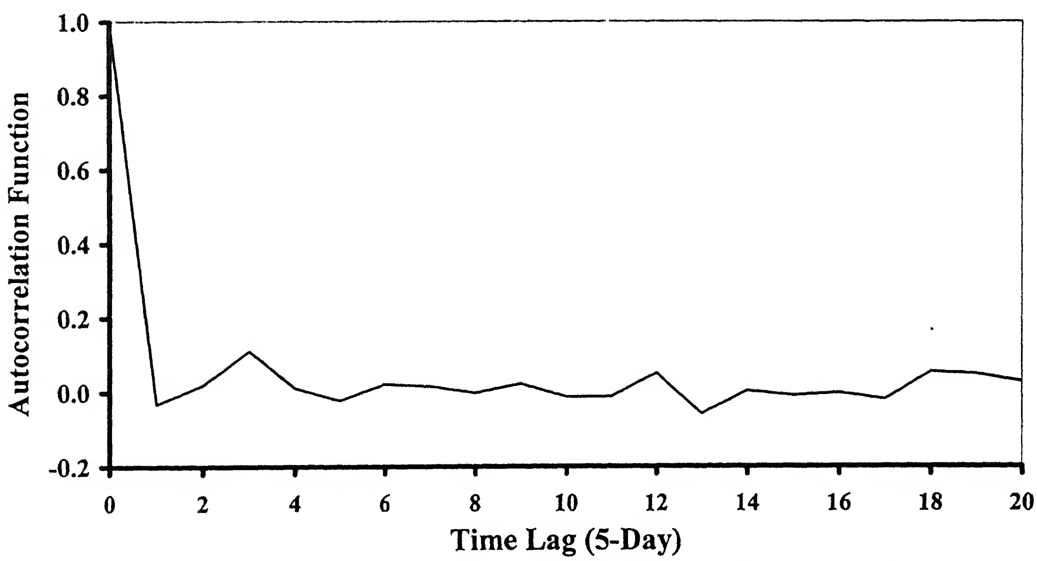


Figure B.10: Correlogram of 5-Day Rainfall at London

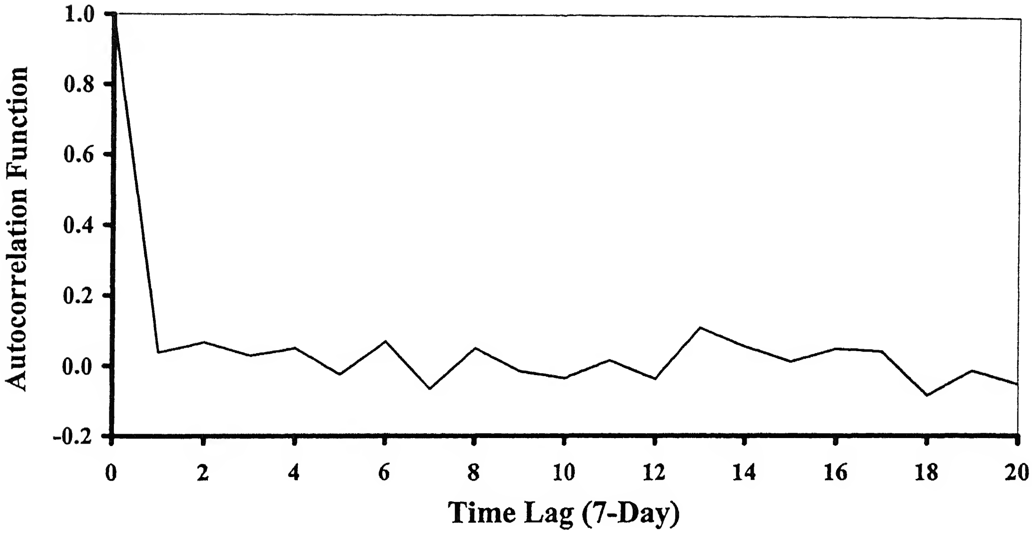


Figure B.11: Correlogram of 7-Day Rainfall at London

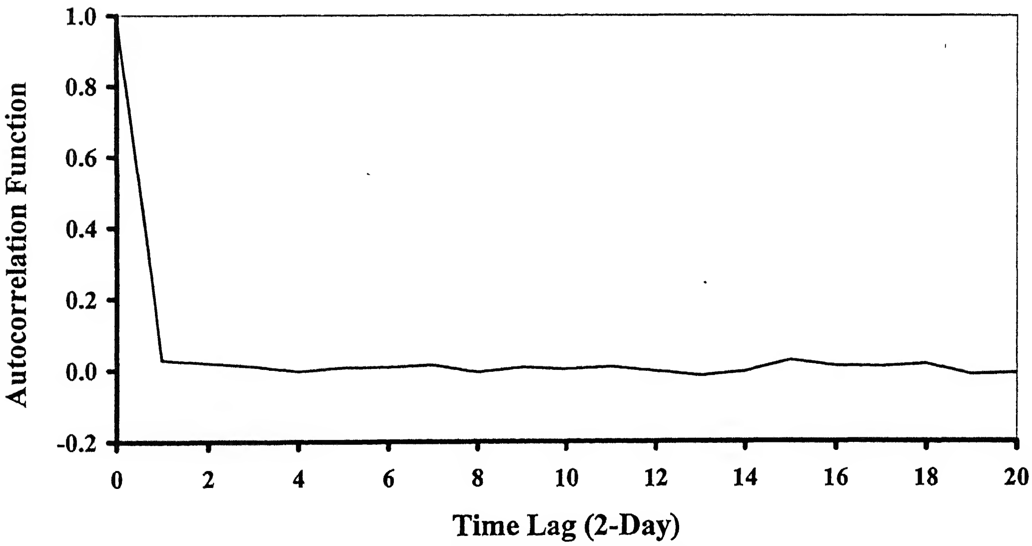


Figure B.12: Correlogram of 2-Day Rainfall at Jackson

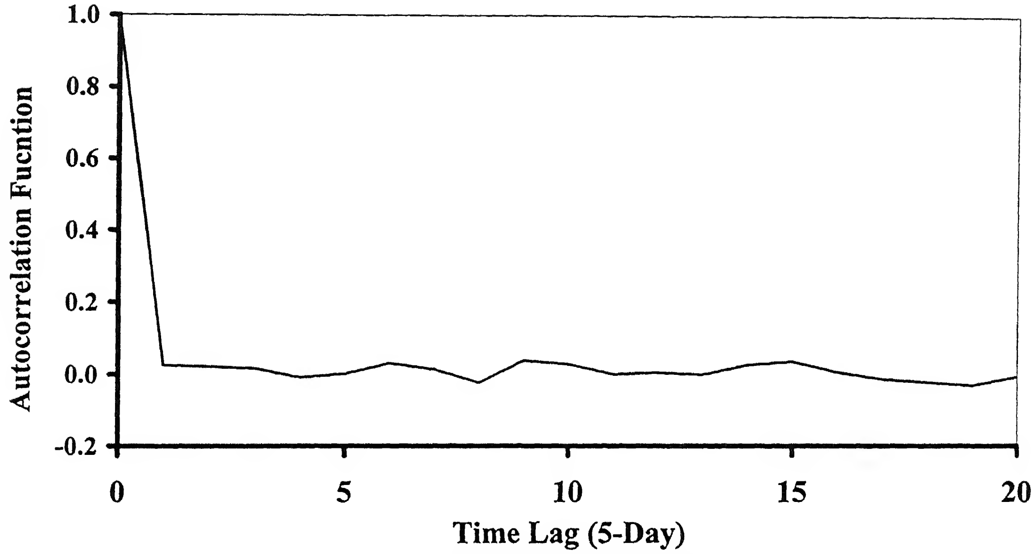


Figure B.13: Correlogram of 5-Day Rainfall at Jackson

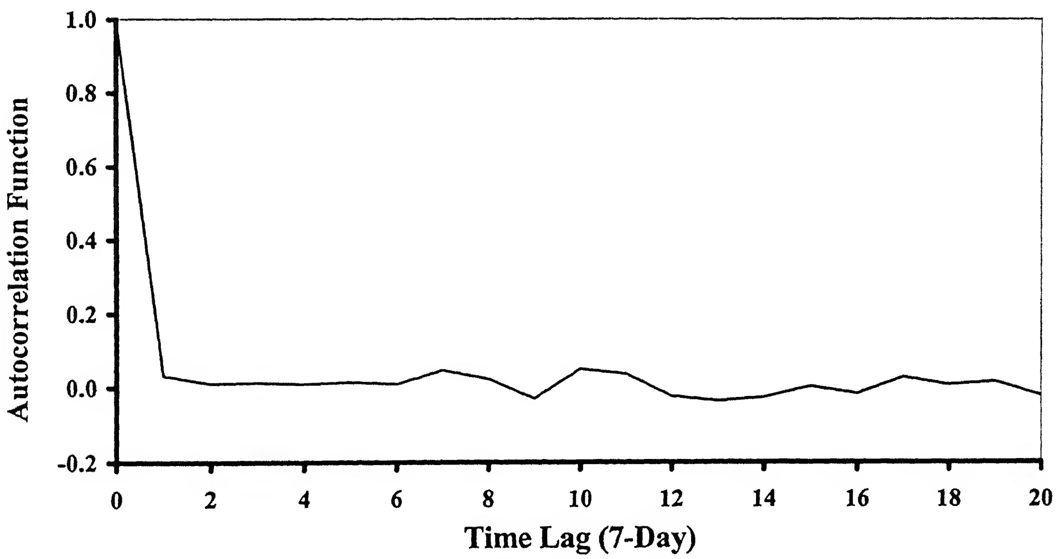


Figure B.14: Correlogram of 7-Day Rainfall at Jackson

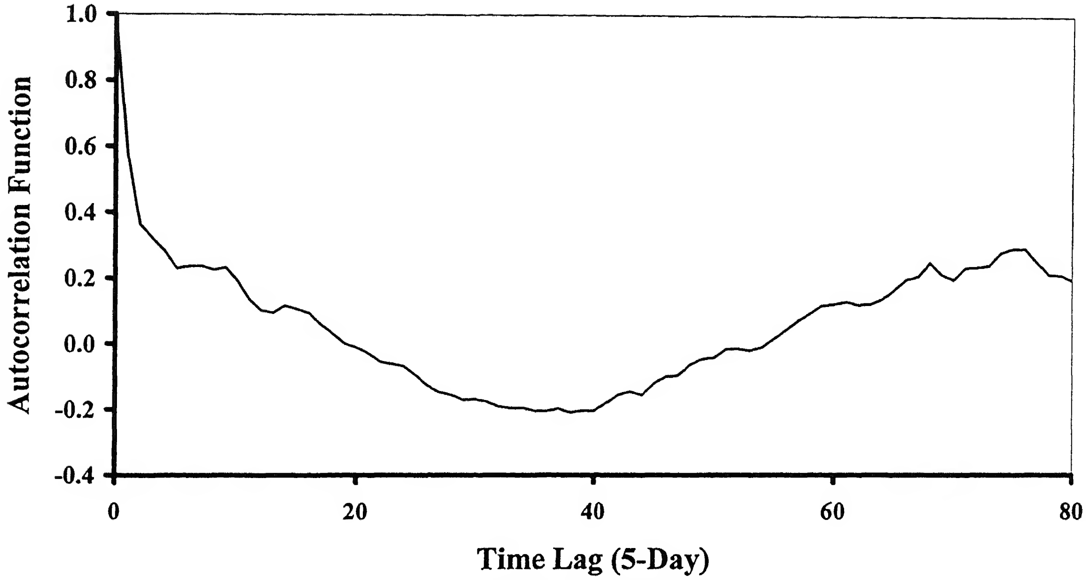


Figure B.15: Correlogram of 5-Day Flow at Lock 10

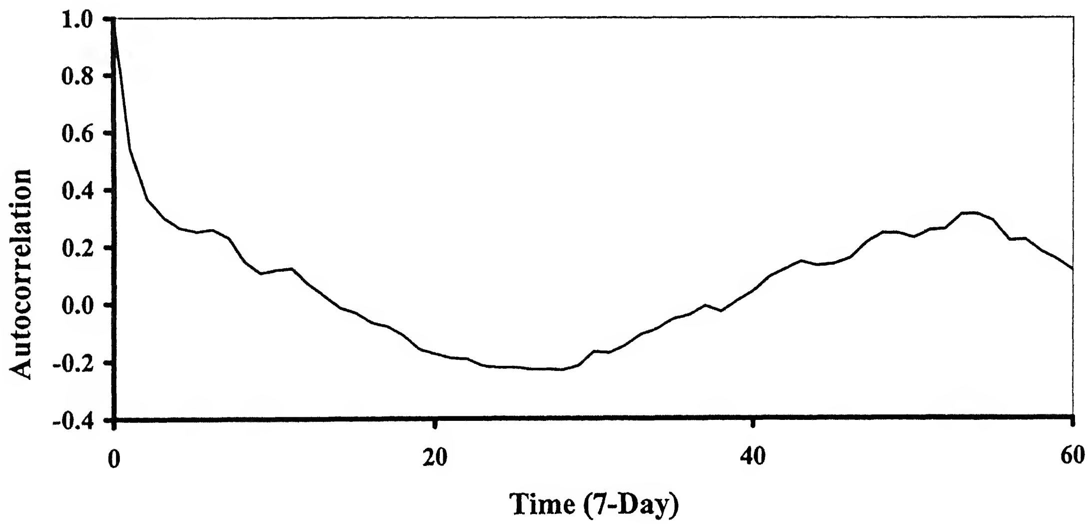


Figure B.16: Correlogram of 7-Day Flow at Lock 10

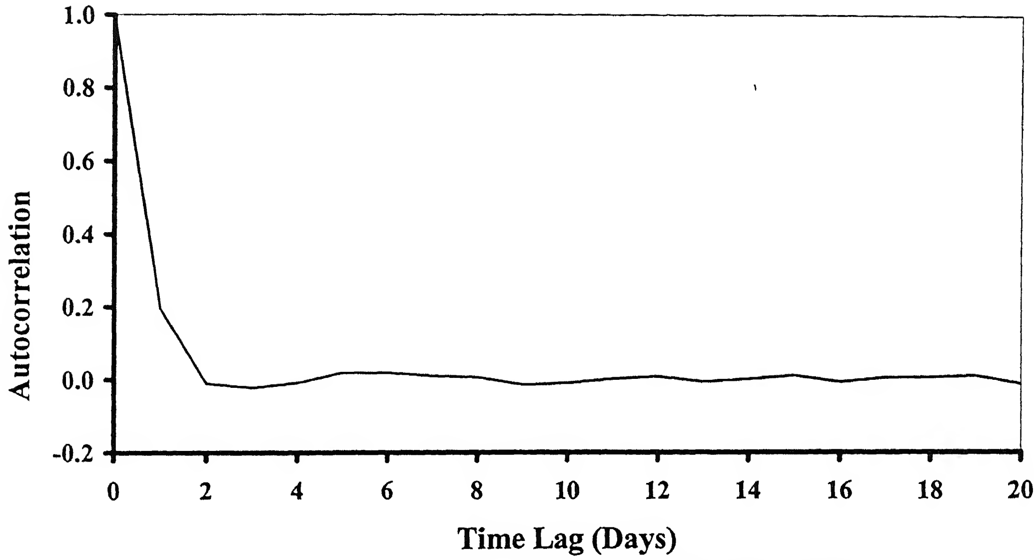


Figure B.17: Correlogram of Average Daily Rainfall at upstream

APPENDIX C

TABLE A.1 Maximum Lyapunov Exponent of Daily Rainfall data at London

m	9	10	11	12	13	14	15	16	17
λ (10^{-2}) (bits/day)	0.228	0.203	0.174	0.164	0.135	0.144	0.109	0.117	0.122
Evolution Time (days)	41	47	53	43	89	49	47	74	79

TABLE A.2 Maximum Lyapunov Exponent of Daily Rainfall data at Jackson

m	9	10	11	12	13	14	15
λ (10^{-2}) (bits/day)	0.190	0.178	0.176	0.163	0.170	0.179	0.169
Evolution Time (days)	63	57	87	76	64	58	60

TABLE A.3 Maximum Lyapunov Exponent of Daily Rainfall data at Hyden

m	9	10	11	12	13	14	15	16	17	18
λ (10^{-2}) (bits/day)	0.186	0.198	0.155	0.142	0.133	0.160	0.138	0.160	0.141	0.125
Evolution Time (days)	073	71	70	54	75	87	64	50	64	61

TABLE A.4 Maximum Lyapunov Exponent of Daily Rainfall data at Manchester

m	9	10	11	12	13	14	15	16	17	18
λ (10^{-2}) (bits/day)	0.260	0.237	0.195	0.208	0.217	0.212	0.158	0.155	0.145	0.150
Evolution Time (days)	65	61	46	53	52	49	40	43	51	41

TABLE A.5 Maximum Lyapunov Exponent of Daily Rainfall data at Heidelburg

m	10	11	12	13	14	15	16
λ (10^{-2}) (bits/day)	0.197	0.164	0.162	0.168	0.207	0.175	0.161
Evolution Time (days)	58	52	55	61	54	69	52

TABLE A.6 Maximum Lyapunov Exponent of Daily Rainfall data at Ford Lock 10

m	9	10	11	12	13	14
λ (10^{-2}) (bits/day)	0.231	0.213	0.181	0.171	0.177	0.166
Evolution Time (days)	63	100	75	64	92	68

TABLE A.7 Maximum Lyapunov Exponent of Average Daily flow at Heidelburg

m	1	2	3	4	5	6	7	8	9	10
λ (10^{-2}) (bits/day)	6.284	0.322	0.184	0.168	0.143	0.131	0.152	0.162	0.174	0.165
Evolution Time (days)	58	66	78	54	57	53	62	80	64	50

TABLE A.8 Maximum Lyapunov Exponent of Average Daily flow at Ford Lock 10

m	1	2	3	4	5	6	7	8	9	10
λ (10^{-2}) (bits/day)	4.664	0.247	0.142	0.215	0.230	0.180	0.163	0.176	0.187	0.170
Evolution Time (days)	78	81	70	88	62	69	73	65	65	67

TABLE A.9 Maximum Lyapunov Exponent of 2-Day Rainfall at London

m	8	9	10	11	12	13	14	15	16
λ (10^{-2}) (bits/day)	0.0815	0.0963	0.0719	0.0559	0.0625	0.0627	0.0527	0.0444	0.0421
Evolution Time (days)	77	97	61	88	96	96	85	43	46

TABLE A.10 Maximum Lyapunov Exponent of 5-Day Rainfall at London

m	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
λ (10^{-2}) (bits/day)	0.0563	0.0512	0.042	0.0454	0.0441	0.0426	0.0425	0.0413	0.0308	0.0289	0.0328	0.0331	0.0309	0.0309	0.029	0.0303
Evolution Time (days)	12	7	65	55	100	32	33	52	86	52	50	55	24	29	77	29

TABLE A.11 Maximum Lyapunov Exponent of 7-Day Rainfall at London

m	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
λ (10^{-2}) (bits/day)	0.0607	0.0476	0.0515	0.0507	0.0361	0.0477	0.0411	0.0387	0.0311	0.0308	0.0402	0.0368	0.0383	0.039	0.0367	0.0364	0.0363
Evolution Time (days)	12	43	86	98	82	81	81	70	81	81	83	81	43	43	43	29	29

TABLE A.12 Maximum Lyapunov Exponent of 2-Day Rainfall at Jackson

m	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
λ (10^{-2}) (bits/day)	0.0776	0.077	0.0532	0.0702	0.0617	0.0678	0.0518	0.0407	0.0528	0.0674	0.0458	0.0423	0.047	0.0476	0.0473
Evolution Time (days)	85	28	59	10	76	30	89	5,35	3	3	21	3	29	29	29

TABLE A.13 Maximum Lyapunov Exponent of 5-Day Rainfall at Jackson

m	7	8	9	10	11	12	13	14	15	16	17	18	19	20
λ (10^{-2}) (bits/day)	0.0415	0.046	0.037	0.0482	0.0371	0.0387	0.03	0.0421	0.0309	0.0319	0.03	0.0281	0.0289	0.0297
Evolution Time (days)	68	92	7	88	3,11,33	41	70	19	77	17	21	23	31	36

TABLE A.14 Maximum Lyapunov Exponent of 7-Day Rainfall at Jackson

m	10	11	12	13	14	15	16	17	18	19	20
λ (10^{-2}) (bits/day)	0.0335	0.0298	0.0339	0.0167	0.0171	0.0171	0.0189	0.0207	0.0267	0.0232	0.0225
Evolution Time (days)	76	18	2	49	49	11,55	3	4	2	7	48

TABLE A.15 Maximum Lyapunov Exponent of 5-Day Flow at Lock 10

m	6	7	8	9
λ (10^{-2}) (bits/day)	0.274	0.193	0.195	0.166
Evolution Time (days)	55	47, 58	24, 28	43

TABLE A.16 Maximum Lyapunov Exponent of 7-Day Flow at Lock 10

m	7	8	9	10
λ (10^{-2}) (bits/day)	0.285	0.296	0.387	0.407
Evolution Time (days)	18	32	9	9

TABLE A.17 Maximum Lyapunov Exponent of Average Daily Rainfall at upstream

m	8	9	10	11	12	13	14	15	16
λ (10^{-2}) (bits/day)	0.161	0.143	0.157	0.131	0.149	0.098	0.102	0.102	0.135
Evolution Time (days)	75	45	41	57	46	56	44	38	46

GLOSSARY

Attractor: If the trajectories converge to a single sub space regardless of the initial conditions, then, it is called an attractor. Such systems are dissipative implying that the energy is not conserved. An attractor can be multi-dimensional, and lies in an *m*-dimensional phase space but has a dimension less than *m*.

Belousov-Zhabotinskill Reaction: It is the prime example of oscillating chemical waves. This reaction is similar to those mechanisms observed in metabolic pathways and some biological processes. It is one of the closest analogs to a life form known in chemistry, showing growing, changing colored concentric rings, which look like a bacterial culture. The Belousov-Zhabotinskii reaction is a chemical reaction that gives rise to a two-dimensional wave front.

Chaotic System: Chaotic systems, which look apparently random and complex, but are of high order non-linear deterministic systems and very sensitive to initial conditions.

Correlation Dimension: Correlation Dimension is a method of investigating the existence of chaos in a time series. The correlation dimension method uses the correlation integral or function (Grassberger and Procaccia 1983) for differentiating chaotic behavior and stochastic behavior.

Couette-Taylor Flow: It is the motion of a fluid between coaxial cylinders experiencing relative motion. Change in the motion of fluid is obtained by varying the torque on the

inner cylinder. During the transition of the flow from laminar to turbulent then flow shows chaotic behavior with a strange attractor.

Diamond Norm: The diamond norm is defined as the sum of all the absolute differences of the elements.

Dynamical System: A dynamical system can be described by a phase-space diagram whose trajectories describe its evolution from some initial state, which is assumed to be known.

Euclidian Norm: The Euclidian norm is defined as the usual way to calculate the distance between two points.

False Nearest Neighbors Technique: It will compute the minimum embedding dimension required for the phase space of a time series or a physical process. It is also known as nearest neighbors technique.

Henon Attractor: Henon attractor generated from the two non linear differential equations. It is also a strange attractor.

Lorenz Attractor: In 1961, Lorenz Edward, a meteorologist working on the weather forecasting with twelve differential equations using computer. To see the same output again, he gave input as middle of the previous outcome in order to save time.

Surprisingly, the generated pattern differentiated from the previous pattern. And later he got different outcomes for slight change of input values. This can be considered as sensitive to initial conditions. Later he worked on the simulation of the simplified model of convection to see this behavior. With a slight change in initial conditions the trajectories took a complete different path and settled as double spiral. The generated trajectories never repeated the same and looked in the random fashion. This attractor is known as Lorenz attractor and equations known as Lorenz equations.

Kolmogorov Entropy: Another method for investigating the existence of chaos in a time-series. Entropy is a thermo dynamic quantity describing the amount of disorder in the system. It can characterize the amount of information needed to predict the next measurements with a certain precision. The Kolmogorov entropy of a time series gives a lower bound to the sum of the Lyapunov exponents. It will have zero value for a regular system (such as periodic systems), positive for chaotic systems and infinite for chaotic systems. It can be calculated from the set of correlation functions (Grassberger and Procaccia, 1983c).

Lyapunov Exponent: The Lyapunov Exponent is another method of investigating for the existence of chaos in a time-series. It measures the average exponential of divergence or convergence of the nearby trajectories in phase space. A system having at least one positive Lyapunov exponent can be defined as the chaotic system.

Mackey-Glass Delay Differential Equation: It is used as a model for the production of the white blood cells. Changing the delay time will produce the chaotic behavior.

Maximum Norm: The maximum norm is just the maximum absolute difference between the elements.

Method of Surrogate Data: This method determines whether the given data set is coming from non-linear process or linear process. The method of surrogate data makes use of the substitute data generated in accordance to the probabilistic structure underlying the original data and null hypothesis. The null hypothesis consists of a candidate linear stochastic process and the objective is to reject the hypothesis that the original data have come from a linear stochastic process. By rejecting this null hypothetic process this method supports the presence of non-linearity in the original data.

Non-Linear Prediction Method: This method quantifies the chaos in the highly complex process and able to forecast the values of the same.

Phase Space: In the phase space representation, system will be represented as function of the values of the process itself unlike the function of the parameters controlling the dynamics of the process.

Rossler Attractor: Rossler system is credited to Otto Rossler and arouse from work in chemical kinematics. The system is generated from three coupled non-linear differential

equations. The trajectories generated by this system never converged to a fixed point or formed limit cycles. The generated trajectories are aperiodic and random and formed attractor known as Rossler attractor.

Scaling of Flow: When the properties of the hydrologic processes are assumed to be independent of the scale (period) of observation then the process is said to exhibit "scaling" or "scaling invariance." The type of such a scaling region is, in general, dependent upon the behavior of the process at different scales under consideration. The transformation of flow data from one scale to another scale (temporal) for e.g. transformation of daily flow to 5-days or 7-days flow can be considered as scaling of flow.

Stochastic Process: It is a phenomenon, in which outcome is neither fully predicted nor truly independent of the occurrence (random). Out comes of these processes are valid in the statistical sense only.

Strange Attractor: In some systems, the trajectories may not converge to a point or to a cycle trajectory. The process never exactly repeats itself. This type of attractor is very peculiar one: it is low dimensional, being contained in a reduced portion of the low-dimensional phase space (e.g., a rectangle, a sphere or box, a hyper sphere), never crosses itself, and never repeats itself. To fulfill these conditions, the attractor has to be an infinite long line within a finite space, and given that it is not a cycle trajectory, will have

non-integer dimension. Such attractors are called ***strange attractors***, and the systems that contain strange attractors are called 'chaotic dynamic systems'.

Trajectory: Trajectories can be thought of, as parametric functions of a variable that is something like time.

Transformation of Data: In a temporal context, the word "transformation" generally refers to the "disaggregation" of low-resolution runoff data to high-resolution ones, since low-resolution data are usually available. In a spatial context, however, it may refer either to the "enlargement" of data from smaller to larger scales or to the "reduction" of data from larger to smaller scales.

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